Specialisation of Attack Trees

with Sequential Refinement

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Three types of refinement:

- Node with undirected arc represents *conjunctive refinement*.
- Node with no arc represents *disjunctive refinement*.
- Node with directed arc represents *sequential refinement*.
Attack Trees Evolve as Domain Knowledge is Specialised

In this specialised tree, “steal backup” can only be performed after breaking into the system.
Criterion for Specialisation of Attack Trees

Criterion:

A specialisation between attack tree is sound with respect to an attribute domain whenever:

valuations are correlated, for any assignment of values to basic actions.

Notes:

▶ “specialisation” and “correlation” have many interpretations.
▶ more general than equality.
Basic minimum attack times:

- bribe sysadmin $\mapsto 25$
- steal backup $\mapsto 5$
- break into system $\mapsto 9$
- install keylogger $\mapsto 2$

$\max\{\min\{25, 5\}, 9+2\} = 11$

$\min\{\max\{25, 9+2\}, 9+\max\{2, 5\}\} = 14$

How do we know: first $\leq$ second for all assignments?
Example: Minimum Number of Experts

Basic number of experts:

- bribe sysadmin $\mapsto 3$
- steal backup $\mapsto 1$
- break into system $\mapsto 2$
- install keylogger $\mapsto 1$

\[
\min\{3, 1\} + \max\{2, 1\} = 3
\]

\[
\min\{3 + \max\{2, 1\}, \max\{2, 1+1\}\} = 2
\]

Valuations correlated, but in opposite direction to previous example.
Trees Correlated Only for Some Domains

- Correlated for “minimum attack time”.
- Uncorrelated for “minimum number of experts”. (Some some valuations ≤ other ≥)
Trees Correlated Only for Some Domains

Uncorrelated for “minimum attack time”. Check assignments:

<table>
<thead>
<tr>
<th>Action</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bribe sysadmin</td>
<td>25</td>
</tr>
<tr>
<td>Steal backup</td>
<td>5</td>
</tr>
<tr>
<td>Break into system</td>
<td>9</td>
</tr>
<tr>
<td>Install keylogger</td>
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<tr>
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<td>9</td>
</tr>
<tr>
<td>Install keylogger</td>
<td>2</td>
</tr>
</tbody>
</table>

Correlated for “minimum number of experts”.
Automating Specialisation

- Even for small examples, *time consuming* and *error-prone* to judge specialisations.
- Unclear what “specialisation” means.
- Better to have tool to check automatically to assist with attack tree manipulation.

Solution define a **semantics** with a **decidable** specialisation relation.
(sound for classes for attribute domain)
Linear Logic in the Sequent Calculus

MALL (Girard 1993):

\[ \frac{\vdash a, a}{\vdash \overline{a}, a} \text{ axiom} \]

\[ \frac{\vdash P, Q, \Delta}{\vdash P \parallel Q, \Delta} \parallel \]

\[ \frac{\vdash P, \Gamma, \Delta}{\vdash Q, \Delta} \otimes \]

\[ \frac{\vdash \Gamma, \Delta}{\vdash \Gamma, \Delta} \text{ mix} \]

\[ \frac{\vdash P_i, \Delta}{\vdash P_1 \oplus P_2, \Delta} \oplus, \quad i \in \{1, 2\} \]

\[ \frac{\vdash P, \Delta}{\vdash Q, \Delta} \land \]

Linear negation defines de Morgan dualities:

\[ \overline{P \parallel Q} = \overline{P} \otimes \overline{Q} \]

\[ \overline{P \otimes Q} = \overline{P} \parallel \overline{Q} \]

\[ \overline{P} \land Q = \overline{P} \oplus Q \]

\[ \overline{P} \oplus Q = \overline{P} \land \overline{Q} \]

\[ \overline{a} = a \]

Linear implication (not P or Q):

\[ P \vdash Q = \overline{P} \parallel Q \]
A Semantics Refining the Multi-set Semantics for Attack Trees

Attack trees related by specialisation:

Proof in sequent calculus:

\[
\begin{align*}
Axiom: & \quad \text{intercept email, intercept email} \\
& \quad \text{\quad complete test, complete test} \\
\implies & \quad \text{intercept email} \otimes \text{complete test, intercept email, complete test} \\
& \quad \text{\quad complete test, intercept email} \parallel \text{complete test} \\
\implies & \quad \text{intercept email} \otimes (\text{intercept email} \parallel \text{complete test}) \oplus \text{compromise server} \\
\implies & \quad (\text{intercept email} \parallel \text{complete test}) \Rightarrow ((\text{intercept email} \parallel \text{complete test}) \oplus \text{compromise server})
\end{align*}
\]
Extending for Sequentiality in the Calculus of Structures

MAV (Horne 2015) in Calculus of Structures (Guglielmi 2007):

\[ \vdash C\{ I \} \]
\[ \vdash C\{ \overline{\alpha} \parallel \alpha \} \quad \text{atomic interaction} \]
\[ \vdash C\{ (P \parallel R) ; (Q \parallel S) \} \quad \text{seq} \]
\[ \vdash C\{ P \otimes (Q \parallel R) \} \quad \text{switch} \]

\[ \vdash C\{ P_i \} \]
\[ \vdash C\{ P_1 \oplus P_2 \} \quad \text{choice} \]
\[ \vdash C\{ (P \parallel R) \& (Q \parallel R) \} \quad \text{external} \]
\[ \vdash C\{ (P \parallel R) & (Q \parallel S) \} \]
\[ \vdash C\{ (P ; Q) \& (R ; S) \} \quad \text{medial} \]
\[ \vdash C\{ I \} \]
\[ \vdash C\{ I \& I \} \quad \text{tidy} \]
\[ \vdash I \quad \text{axiom} \]

commutative monoids: \((P, \parallel, I)\) \((P, \otimes, I)\)

monoid: \((P, ; , I)\)

de Morgan dualities:
\[ \overline{P \otimes Q} = \overline{P} \parallel \overline{Q} \]
\[ \overline{P \parallel Q} = \overline{P} \& \overline{Q} \]
\[ \overline{P ; Q} = \overline{P} ; \overline{Q} \]
\[ \overline{\alpha} = \alpha \]
\[ \overline{I} = I \]
Example Verified using the Calculus of Structures

The first tree specialises (implies) the second.

Proof:

\[
\begin{align*}
& \vdash (bribe \parallel bribe) \otimes ((breakin \parallel breakin) ; (install \parallel install)) \land ((breakin \parallel breakin) ; ((steal \parallel steal) \otimes (install \parallel install))) \\
& \vdash (bribe \parallel bribe) \otimes ((breakin \parallel breakin) ; (install \parallel install)) \land ((breakin \parallel breakin) ; ((steal \parallel install) \parallel steal \parallel install)) \\
& \vdash ((bribe \parallel breakin) \otimes (breakin ; install) \parallel (breakin ; install)) \land ((breakin ; (steal \parallel install)) \parallel steal \parallel (breakin ; install)) \\
& \vdash (bribe \parallel (breakin ; install)) \parallel (bribe \parallel steal) \parallel (breakin ; install) \land ((bribe \parallel steals) \parallel (breakin ; install)) \\
& \vdash (bribe \parallel (breakin ; install)) \& (breakin ; (steal \parallel install)) \parallel (bribe \parallel steal) \parallel (breakin ; install) \\
& \vdash (bribe \parallel (breakin ; install)) \& (breakin ; (steal \parallel install)) \parallel (bribe \parallel steal) \parallel (breakin ; install) \\
& \vdash (bribe \parallel (breakin ; install)) \parallel (bribe \parallel steals) \parallel (breakin ; install) \\
& \vdash (bribe \parallel (breakin ; install)) \parallel (bribe \parallel steals) \parallel (breakin ; install) \\
& \vdash (bribe \parallel (breakin ; install)) \parallel (bribe \parallel steals) \parallel (breakin ; install) \\
\end{align*}
\]
Relates Trees Unrelated by Related Semantics for Causal Attack Trees

Trees Related by Specialisation (but not by set inclusion in Jhawar et al. 2015):

Extra causal dependencies clear in graphical model (adapted from Gischer 1988):
Subtleties: Partial Distributivity

Trees equivalent for Jhawar et al. 2015.

..but specialisation holds in one direction only according to MAV.

“Operational” explanation: The “local” disjunctive refinement allows choices to be delayed ...permits less coordination between sub-goals.
Perspectives on attack trees

- **Non-deterministic v.s. probabilistic choice:** *minimum* time selects best case choices; *maximum* time selects worst case; *expected* time involves a contribution from all branches; hence projection forbidden.

- **Attack-defence trees:** Semantics lifts to specialisation for attack-defence trees respecting multi-sets semantics (that assumes attacker resolves all choices).

- **Breaking asymmetry:** Does the attacker always have control of choices made during an attack? E.g. can the attacker actively chose whether it is killing a master node or data node in the following (the defender may pro-actively conceal the master node).

```
  impair availability
   / \            /  \\
  disrupt network kill node
   / \            /  \\
kill master node kill data node
```

- **Provenance and fault diagrams:** provenance diagrams (origin of MAV), fault diagrams (“safety” countermeasures suggest exploitable vulnerabilities)... there are common foundations and applications.
Conclusion

- **Specialisation** useful for comparing attack trees that are **not necessarily equal**.

- Semantics for specialisation depends on **class of attribute domain**:
  - One class illustrated by “minimum attack time”;
  - Another class illustrated by “minimum number of experts”.

- **Semantics** for each class provided by embedding in logical system MAV.

- Specialisation is **decidable**. ...leading to support in ADTool?

- ...but does the attacker always have control of choices?