Design of Antiresonant-Reflecting Optical Waveguide-Type Vertical-Cavity Surface-Emitting Lasers Using Transfer Matrix Method

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***Abstract*—A simple transfer matrix method is proposed to calculate the radiation losses of a cylindrical antiresonant- reflecting optical waveguide (ARROW). It is found that the planar approximation underestimates the radiation losses but overestimates the design tolerance of the ARROW-type vertical-cavity surface-emitting lasers (VCSELs). An iteration technique, which is incorporated with the transfer matrix method, is also developed to design ARROW-type VCSELs for minimum radiation losses in a more effective manner.**

***Index Terms*—Antiresonant-reflecting optical waveguide (ARROW), radiation losses, semiconductor lasers, vertical-cavity surface-emitting lasers (VCSELs).**

I. Introduction

RECENTLY, high-power single-mode operation of vertical-cavity surface-emitting lasers (VCSELs) using cylindrical antiresonant-reflecting optical waveguide (ARROW) has been proposed and demonstrated. This is because ARROW-type VCSELs allow 1) low radiation loss and 2) large *spot-size* single-mode operation. In fabrication of ARROW-type VCSELs, the design-rule of planar ARROW is adapted to design the dimensions of the ring reflectors [1]. However, our more exact calculation shown that the approximation of cosine and sine waves to the Bessel functions (i.e., planar approximation) is not appropriate even for large aperture cylindrical ARROW.

In this letter, a simple transfer matrix method is derived to calculate the radiation losses of a cylindrical ARROW. Our more exact method shows that the planar approximation underestimates the radiation losses but overestimates the design tolerance, especially for the second-cladding region of the ARROW-type VCSELs. An iteration technique is also developed to design ARROW-type VCSELs in a more effective manner.

II. Transfer Matrix Method for Cylindrical ARROW

An ARROW, which is represented by four layers of concentric rings, is shown in Fig. 1. The optical fields, which satisfies the scalar Helmholtz equation in cylindrical coordinate, propagating along the lateral direction r can be written as

$$E_i = E_i^{+} H_v^{(1)}(\beta_i r) + E_i^{-} H_v^{(2)}(\beta_i r) \tag{1}$$

Manuscript received January 2, 2003; revised May 19, 2003.
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Digital Object Identifier 10.1109/LPT.2003.816116

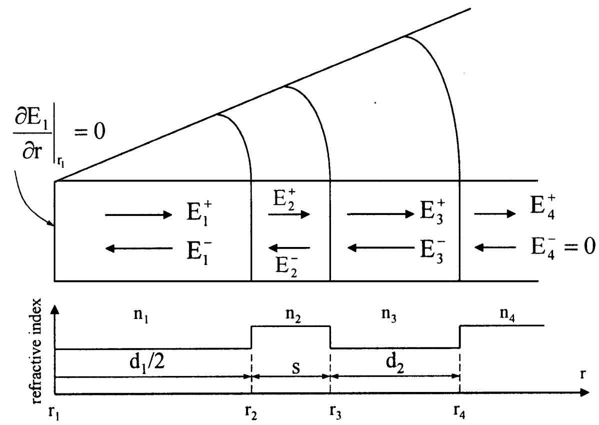

Fig. 1. Schematic diagram of a cylindrical ARROW and equivalent refractive index profile along the lateral direction r. In the calculation, it is assumed that $d_1 = 8$ μm, $n_1 = n_3 =$ 3.3, and $n_2 = n_4 =$ 3.35. The regions between r_1 and r_2, r_2 and r_3, and r_3 and r_4 are defined as the core, first-cladding and second-cladding regions of the ARROW.

where E_i represents the field profile in the ith con-centric ring, E_i^- and E_i^+ are the corresponding in- and out-going traveling field amplitude, respectively. $H_v^{(1)}$ and $H_v^{(2)}$ are the Hankel functions of order v. $\beta_i \left(= 2\pi\sqrt{n_i^2 - n_{\text{eff}}^2}/\lambda\right)$ is the lateral propagation coefficient, λ is the vacuum wavelength, n_i is the refractive index, and n_{eff} is the effective refractive index to be deduced. It can be shown that the electric field E_i and its derivative E_i' can be related to E_{i+1} and E_{i+1}' at the adjacent con-centric ring layer by

$$\begin{aligned} &\begin{bmatrix} E_i \\ E_i' \end{bmatrix}_{r=r_i} \\ &= \begin{bmatrix} H_v^{(1)}(\beta_i r_i) & H_v^{(2)}(\beta_i r_i) \\ H'^{(1)}_v(\beta_i r_i) & H'^{(2)}_v(\beta_i r_i) \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} H_v^{(1)}(\beta_i r_{i+1}) & H_v^{(2)}(\beta_i r_{i+1}) \\ H'^{(1)}_v(\beta_i r_{i+1}) & H'^{(2)}_v(\beta_i r_{i+1}) \end{bmatrix}^{-1} \begin{bmatrix} E_{i+1} \\ E_{i+1}' \end{bmatrix}_{r=r_{i+1}} \\ &\equiv T_i \begin{bmatrix} E_{i+1} \\ E_{i+1}' \end{bmatrix}_{r=r_{i+1}} \end{aligned} \tag{2}$$

where $i = 1, 2,$ or 3 and the boundary conditions [2] have been considered in derivation of (2). The transfer matrix for the entire cylindrical ARROW T can be obtained by the product of the respective transfer matrices T_i of the individual con-centric rings, that is $T = \prod_{i=1}^{3} T_i$. The boundary condition at $r = r_1$ can be obtained by using the symmetric property of the electric fields, that is the field derivative at $r = r_1$ goes to zero (i.e.,

$E'_1 = 0$). The boundary condition at the outer con-centric ring (i.e., $r > r_4$) has no in-going wave (i.e., $E_4^- = 0$) so that we have

$$\begin{bmatrix} E_4 \\ E'_4 \end{bmatrix} = \begin{bmatrix} H_v^{(1)}(\beta_4 r) & H_v^{(2)}(\beta_4 r) \\ H'^{(1)}_v(\beta_4 r) & H'^{(2)}_v(\beta_4 r) \end{bmatrix} \begin{bmatrix} E_4^+ \\ 0 \end{bmatrix}. \quad (3)$$

However, (3) has to be further simplified in order to remove the components that are functions of r. It must be noted that this problem will not occur in planar consideration because the in- and out-going waves are of exponential functions (i.e., planewave). Hence, the r dependence of the electric field and its derivative can be removed so that the electric field and its derivative can be related by a constant [2]. In order to solve this problem, we used the following approximation to the Hankel functions:

$$H_v^{(1)}(\beta_4 r) \approx \sqrt{\frac{2}{\pi\beta_4 r}}\left[\cos\left(\beta_4 r - \frac{\pi}{4} - \frac{1}{2}v\pi\right) + j\sin\left(\beta_4 r - \frac{\pi}{4} - \frac{1}{2}v\pi\right)\right]$$

and $H'^{(1)}_v(\beta_4 r) \approx j\beta_4 H_v^{(1)}(\beta_4 r)$. This approximation is satisfied because $\beta_4 r (> 110) \gg v$ for $v = 0, 1$ and $r > r_4(\sim 9\ \mu\text{m})$. Hence, it can be shown that the field derivation term reduces to $E'_4 \approx j\beta_4 E_4$. Finally, the eigen equation $\eta(n_{\text{eff}})$ for the radiation modes in cylindrical ARROW can be reduced to

$$\eta(n_{\text{eff}}) = t_{21} + j\beta_4 t_{22} = 0 \quad (4)$$

where t_{21} and t_{22} are the elements of matrix T. This proposed transfer matrix method has used an effective index approximation, and hence, the modal behavior may deviate from the actual three-dimensional VCSEL modes obtained from exact calculation [3], even though the error is very small.

III. Results and Discussion

It can be shown that [1, Fig. 5] can be reproduced by planar consideration and the result is given in Fig. 2(a). Our more exact calculation using transfer matrix is also shown in Fig. 2(b). It is observed that the radiation loss calculated by planar approximation (i.e., [1, Fig. 5]) is lower than that obtained from our more exact solution. However, the optimal value of s obtained from the planar approximation is valid as it is only different by 4% (i.e., 0.05 μm) when compared with our more exact calculation. This indicated that the planar approximation used in [1] is a good estimation of s; even the corresponding radiation losses are underestimated.

In the design of the circular ARROW, the planar approximation is usually used to deduce s and d_2. It is suggested that d_2 $(= d_1/2)$ and s should be about $\lambda_l/4$ and $3\lambda'_l/4$ [4], respectively, in order to obtain standing wave profile with nodes at r_2 and r_4. λ_1 and λ'_l are the lateral wavelength at the regions *having* refractive indexes of n_1 and n_2, respectively. Hence, the analysis in [1] gives $s \sim 1.3\ \mu$m and $d_2(= d_1/2) \sim 4\ \mu$m. However, our calculation shows that $d_2 \sim 4\ \mu$m is far away from the optimal value and will not be tolerated in the design of cylindrical ARROW. Fig. 3 shows the computed radiation losses of the fundamental mode using both planar approximation and our more exact transfer matrix method with d_2 as the variable. The other parameters used in the calculation are shown in the figure caption. In the calculation, the planar approximation suggested that

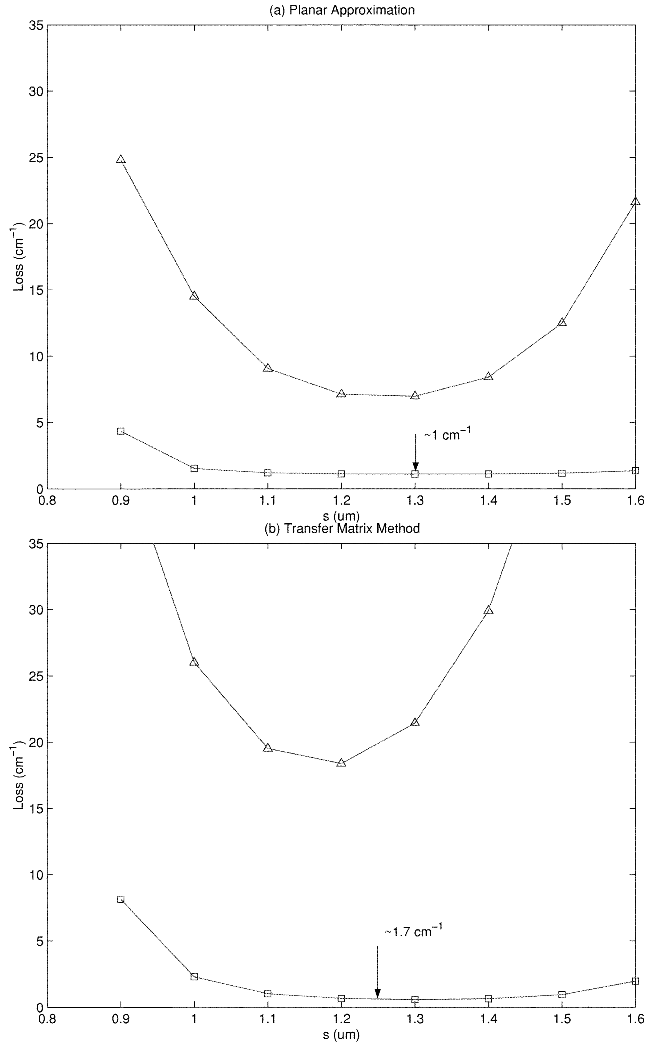

Fig. 2. Modal radiation losses of the two lowest loss modes (□—Mode 4 and △—Mode 7) of ARROW-type VCSEL versus s with $d_1/2 = d_2 = 4\ \mu$m and $\lambda = 0.98\ \mu$m. (a) Computed using planar approximation and matched with the results given in [1, Fig. 5]. (b) Computed using proposed transfer matrix method. The value of s for minimum radiation loss of the fundamental mode is found to be 1.3 and 1.25 μm for the cases computed by planar approximation and transfer matrix method, respectively.

the optimum value of d_2 is about 4 μm but our more exact calculation requires d_2 to be about 2.65 μm. This is because of the nonuniform periodicity of the Bessel function. Inside the core region (i.e., $r_1 \leq r \leq r_2$) of the cylindrical ARROW, the phase of the fundamental mode, which can be calculated from the first root of $J_0(\zeta_1) = 0$, is found to be $\zeta_1 = 2.403$. However, the planar approximation gives $\zeta_1 = \pi/2$, which can be obtained from the first root of $\cos(\zeta_1) = 0$. Inside the second-cladding region (i.e., $r_3 \leq r \leq r_4$), it is noted that both Bessel and cosine functions give $\zeta_1 = \pi/2$ as the period of $J_0(x)$ is close to $\cos(x)$ for a large value of x. Hence, the value of d_2 should be shorter than $d_1/2$ by a factor of 1.53 $(= 2.403/\pi/2)$. If $d_1/2 = 4\ \mu$m, d_2 should about $1.53^{-1} \times d_1/2 = 2.61\ \mu$m, which is quite close to our numerical value. For the cylindrical ARROW, it is noted that for a fixed value of d_1, there should has a combination of s and d_2 for minimum radiation loss. Fig. 4 plots the minimum radiation losses with different combinations of s and d_2 for a device with $d_1 = 8\ \mu$m. It is observed that when s increases from 0.95 to 1.4 μm and d_2 reduces from $\sim$ 3 to $\sim$ 2.62 μm.

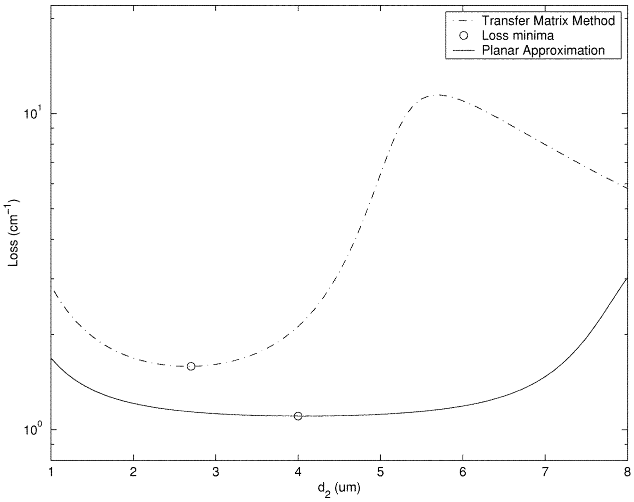

Fig. 3. Modal radiation losses of the fundamental mode of ARROW-type VCSELs versus d_2 computed by planar approximation and transfer matrix method. In the calculation, $d_1 = 8$ μm, $\lambda = 0.98$ μm, and s is set to 1.3 and 1.25 μm for the cases using planar approximation and transfer matrix method, respectively. It is shown that the optimal value of d_2 (labeled "o") is $\sim$ 4 and $\sim$ 2.65 μm for the cases computed by planar approximation and transfer matrix method, respectively.

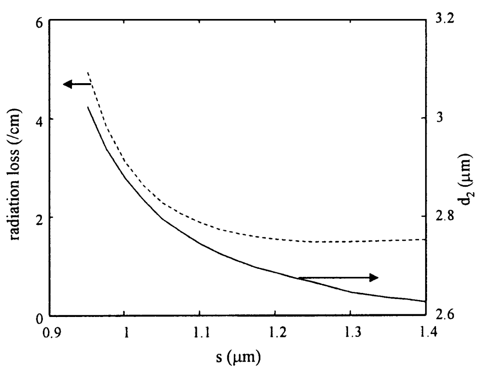

Fig. 4. Variation of optimal d_2 and radiation losses versus s of a cylindrical ARROW with $d_1 = 8$ μm. The other parameters used in the calculation are the same as in Fig. 3.

The reduction of d_2 is required in order to maintain the node of the electric field at r_4 for minimum radiation losses. The radiation loss has a global minimum, which occurs at $s \sim 1.25$ μm and $d_2 \sim 2.65$ μm.

A more rigorous design method is also developed, using an iteration technique to incorporate with the transfer matrix method, to calculate the optimal dimensions of the cylindrical ARROW in a more effective manner. Fig. 5 illustrates the calculation process of the iteration technique. At the beginning of the calculation, λ, d_1, and the refractive indexes n_1 to n_4 are the parameters to be given by the users. In addition, the values of s and d_2 are estimated by a guessed n_{eff} of the ARROW. All the parameters are then substituted into the transfer matrix to solve for a new n_{eff}, which will then be used to find the value of $\partial E_2/\partial r$ at r_3 and E_3 at r_4. If the value of $\partial E_2/\partial r < 0$ ($E_3 < 0$), r_3 (r_4) will be reduced by a small amount $\delta(\varepsilon)$. However, if $\partial E_2/\partial r > 0$ ($E_3 > 0$), r_3 (r_4) will be increased by a small amount $\delta(\varepsilon)$. After the adjustment, r_3 and r_4 will be substituted back into the transfer matrix for another n_{eff}. The

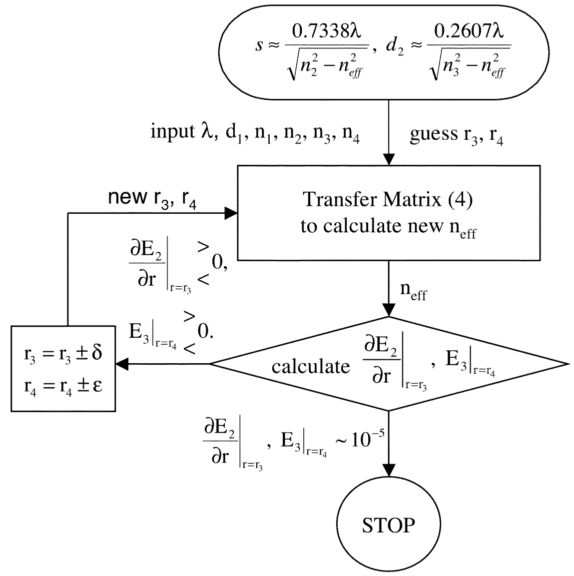

Fig. 5. Flowchart of the iterative technique to calculate the exact dimensions of the cylindrical ARROW with minimal radiation losses.

iteration process stops when the absolute values of $\partial E_2/\partial r$ and E_3 are both less than 10^{-5}. This algorithm deduces the values of r_3 and r_4 according to the positions of antinode and node of the electric field. This is because minimum radiation loss occurs when the antinode and node of the electric field are coincided with r_3 and r_4, respectively. Using the algorithm, it is found that $s \sim 1.25$ μm and $d_2 \sim 2.65$ μm, for the cylindrical ARROW with $d_1 = 8$ μm, which are consistent with Fig. 4. The corresponding computational time is less than a second when the MATLAB program runs on a Pentium III 500-MHz personal computer.

IV. Conclusion

A simple transfer matrix method is proposed to calculate the radiation losses of a cylindrical ARROW. An iteration technique is also suggested to optimize the dimension of ARROW-type VCSELs for minimum radiation losses in a more effective manner. The main advantage of the transfer matrix method is that the nonuniform distribution of refractive index inside the cylindrical ARROW can also be analyzed. Hence, this method can be applied to study the nonuniform distribution of refractive index inside the active layer of VCSELs arisen from spatial-hole-burning or thermal lensing.

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