Variable Thresholds in the Chaotic Cellular Neural Network

Wen Liu and Lipo Wang

Abstract—The chaotic cellular neural network (C-CNN) has complex dynamics, including chaos, oscillations, and stable fixed points. Chaotic dynamics can help the network avoid local minima and reach the global optimum. Hence chaos can improve the performance of cellular neural networks (CNNs) on problems that have local minima in energy (cost) functions. We investigate the effect of variable thresholds in the C-CNN. We show that this threshold cannot be too large if one wishes to produce chaotic dynamics in the C-CNN, which is important to studies of chaotic communication and combinatorial optimization problem. We are particularly interested in variable thresholds because Shi and Wang [1] showed that the objectives of the frequency assignment problem (FAP) can be mapped into thresholds of the neural network, which resulted in superior performance compared to traditional penalty approaches.

I. INTRODUCTION

The cellular neural network (CNN) introduced by Chua and Yang [2] has gained much interest in recent years due to its local connectivity and easy hardware implementation. Many impressive applications of CNNs to image processing have been presented in the literature [3], [4], spanning widely from noise removal, ancient Chinese character recognition, to weather forecasting. The mature VLSI technology for CNN implementations also inspires the development of artificial vision systems [5], [6]. CNNs have some significant merits, such as continuous time network output and parallel processing so that computation time is independent of the problem size [7]. However, with monotonous network dynamics, CNNs may not be able to manage some complex problems.

Several papers have studied the stability of CNNs, including oscillation and chaotic behaviors [8]–[10]. We presented a chaotic cellular neural network (C-CNN) [11] by introducing negative self-feedback into the Euler approximation of the continuous CNN. This new model has various dynamics including limit cycles and chaotic attractors depending on the magnitude of the self-feedback. In this paper, we explore how the threshold (bias) in the self-feedback term affects the dynamics of the C-CNN model. Shi and Wang [1] showed that thresholds of the noisy chaotic neural network (NCNN) [12] can be related to the objective of an optimization problem. They defined the threshold so that the neuron with a smaller cost is more likely to be chosen for frequency assignments in satellite communication systems. Hence variable thresholds separate the optimization objective term from the constraint terms in the energy function and helps the network model to reach the optimal solution. They showed that NCNN with variable thresholds out-performed traditional penalty approaches.

Nonlinear dynamics is applied in various disciplines, such as biology, economics, physiology, and engineering. Andreyev et al [13] explored information processing capabilities of chaos and complex dynamics. They found that a model with chaotic dynamics is efficient in storage, retrieval, and recognition of information and has low computation cost.

Aihara [14] has argued that chaotic dynamics has many possible useful functions. To model dynamics of the biological brain, more complex system is needed. Aihara [14] concluded that synchronous and asynchronous models with complex dynamics may provide parallel distributed processing which will be complementary to the present digital computers.

This paper is organized as follows. We review the C-CNN model in Section 2. Simulation results on the effect of thresholds are presented in Section 3. Finally, we conclude this paper in Section 4.

II. THE CHAOTIC CELLULAR NEURAL NETWORK

A block diagram of the C-CNN model [11] is shown in Figure 1. The network dynamics is described by the following differential equations:

\[
\frac{dx_{ij}(t)}{dt} = -x_{ij}(t) + \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l) y_{kl}(t) + \sum_{C(k,l)\in N_r(i,j)} B(i,j;k,l) u_{kl} - z \left[ y_{ij}(t) - I_0 \right] + I
\]

\[
y_{ij}(t) = \frac{1}{2} (|x_{ij}(t) + 1| - |x_{ij}(t) - 1|)
\]

where \(x_{ij}\) and \(y_{ij}\) denote the internal state and output of neuron \((i, j)\), respectively. \(u_{ij}\) is the input of neuron \((i, j)\) from an independent voltage source. \(A(i, j; k, l)\) and \(B(i, j; k, l)\) are the output feedback parameter and the input control parameter, respectively. \(C(i, j)\) denotes the cell \((i, j)\) and \(N_r(i, j) = \{(k, l) | \text{max}(|k - i|, |l - j|) \leq r, 1 \leq k \leq M, 1 \leq l \leq N\}\) is the r-neighborhood of neuron \((i, j)\). Figure 2(a) shows an example of the 1-neighborhood cells in a 2-dimensional 4 × 4 CNN. The nine gray cells are the r-neighborhood cells of the black cell \(C(i,j)\) when \(r = 1\). Figure 2(b) shows the piecewise-linear activation (output) function of the C-CNN as in equation (2). Besides, \(I\) is an independent voltage source, \(z\) is the self-feedback connection weight, and \(I_0\) is a positive bias (\(-I_0\) being the firing threshold) which we are going to study in this paper.

Wen Liu and Lipo Wang (corresponding author) are with College of Information Engineering, Xiangtan University, Xiangtan, Hunan, China and School of Electrical and Electronic Engineering, Nanyang Technological University, Block S1, 50 Nanyang Avenue, Singapore 639798. (phone: +65 6790 6372; fax: +65 6793 3318; email: {liuw0004, elpwang}@ntu.edu.sg).
III. SIMULATION RESULTS

For the C-CNN, $I_0$ may affect chaotic dynamics. We simulate a single-neuron model of the C-CNN by employing the first order Euler approximation with $\Delta t = 0.0001$. From (1), the first order Euler approximation of a single-neuron model is:

$$x(t + 1) = kx(t) - z[y(t) - I_0] + Ay(t) + Bu + I$$

where $k = 1 - \Delta t$. In the simulation, the initial value for neuron state $x(0)$ is randomly generated in $[-1, 1]$ and the other parameters are set to [11]:

$$A = 1, \quad Bu = 0, \quad I = 0, \quad z = 5.$$ 

Lyapunov exponent [15] is computed to identify the deterministic chaos, which is defined in equation (4). For each fixed value of $I_0$ from 0.01 to 1 with an interval 0.01, we compute the Lyapunov exponent of the C-CNN dynamics with equations (2), (3), and (4) for sufficiently large iterations ($m = 10000$).

$$\lambda = \lim_{m \to +\infty} \frac{1}{m} \sum_{t=0}^{m-1} \ln \left| \frac{dx(t + 1)}{dx(t)} \right|$$

Considering that the activation function, i.e., equation (2), can be written as:

$$y(t) = \begin{cases} 
1 & 1 < x(t) \\
-x(t) & -1 \leq x(t) \leq 1 \\
-1 & x(t) < -1 
\end{cases}$$

From equations (3) and (5) we have:

$$\frac{dx(t + 1)}{dx(t)} = \begin{cases} 
k & 1 < x(t) \\
k - z + A & -1 \leq x(t) \leq 1 \\
k & x(t) < -1 
\end{cases}$$
Lyapunov exponents on different thresholds settings are shown in Figure 3. For \( I_0 > 0.8 \), \( \lambda = -0.0089 \), which implies the system exhibits fixed points [16]. For \( I_0 < 0.8 \), \( \lambda \) is mostly positive, which means the network dynamics is chaotic [16]; however, there exist periodic windows with negative \( \lambda \), for example, \( I_0 = 0.27 \) (period-3). We find that we should have \( I_0 < [z - A(i,j; i,j) - I]/z \) in order to produce chaos, i.e., \( I_0 < 0.75 \) if \( z = 4 \), though without proof.

We further observe the output of the neuron for fixed \( I_0 \). Figure 4 shows neuron output \( y \) as a function of time \( t \) in the single-neuron C-CNN, for different choices of \( I_0 \) (but fixed against time \( t \)). For each \( I_0 \), we present the output in 40 iteration steps with randomly generated initial states. Figure 4(a) shows a chaotic output when \( I_0 = 0.18 \). In Figure 4(b) with \( I_0 = 0.27 \), the neuron output begins oscillating between 1, 1, and \(-1\) (period-3 oscillations) - “Period-3 implies chaos” [17]. When \( I_0 = 0.8 \), the network approaches a fixed point.

We now let \( I_0 \) decay exponentially, i.e., \( I_0(t + 1) = (1 - \beta)I_0(t) \), where \( \beta \) is the decaying factor. We start from the initial value \( I_0(0) = 1 \) and \( \beta = 0.0015 \). The internal state and output of the neuron are shown in Figure 5 (a) and (b). Consistent with the result in Figure 3, in the first about 150 iterations, i.e., when \( I_0 > 0.8 \), there is only one stable equilibrium for the output. Chaos emerges as \( I_0 \) goes smaller.

Different applications may require different durations of chaotic dynamics (determined by the damping factor \( \beta \)) or different types of damping scheme (we use exponential damping in this paper). These parameters are required to be set properly corresponding to the user requirement.

IV. CONCLUSION

We have shown previously [11] that the C-CNN model with a variable neuronal self-feedback is able to produce various dynamics, including chaos, periodic oscillations, and stable fixed points. In this paper, we have shown that variable thresholds in the C-CNN model has a great effect on producing chaos. Chaos exist when the threshold is small and there is no chaotic dynamics if the threshold \( I_0 > [z - A(i,j; i,j) - I]/z \). In the future, we will further explore the C-CNN model, with applications in chaotic communications and combinatorial optimization problems [18], [19].

Cellular neural networks with chaotic dynamics may also help in the development of image processing techniques. Generally, the CNN converges to only binary outputs of \(-1\) and \(1\) according to network dynamic functions with simple feedback and control templates. Hence only a fully saturated or a black color can be obtained from the CNN [20]. With complex nonlinear dynamics, the C-CNN has potential in applications of grey images and even color images.

REFERENCES

Fig. 5. The internal state and output of a single-neuron chaotic cellular neural network with an exponential decaying $I_0$ (a) the internal state of neuron $x$; (b) the output of neuron $y$; (c) exponential decaying $I_0$. 


