Midterm exam MAS436, Sept 2008

Question 1. (25 marks (out of 100 total))
Let X, Y be topological spaces and $A, B \subset X$ two closed sets such that $X = A \cup B$. Let $f : X \to Y$ be a function. Show that if the restrictions $f|_A$ and $f|_B$ are continuous then f is continuous.

Question 2. (25 marks)

Let X be a topological space and $X \times X \supset A = \{(x, x) \mid x \in X\}$. Show that X is Hausdorff iff A is closed in $X \times X$.

Question 3. (25 marks)

Define the topology on $X := \mathbb{R} \cup \{\infty\}$, where ∞ is a new symbol, as follows: the open sets are the sets $U \subset \mathbb{R}$ open in the usual topology of $\mathbb{R}$, and these of the form $U = X \setminus C$, for any compact $C \subset \mathbb{R}$.

- Show that X is compact.
- Construct a homeomorphism from the unit circle to X.

Question 4. (25 marks)
Let (M, d) be a metric space and $\{a_n \mid n = 1, 2, \dots\}$ a countable dense subset of M. Define

$$f : M \to \mathbb{R} \times \mathbb{R} \times \dots$$
$$a \mapsto (d(a_2, a) - d(a_2, a_1),\ d(a_3, a) - d(a_3, a_1), \dots, d(a_{n+1}, a) - d(a_{n+1}, a_1), \dots)$$

Show that f is a homeomeorphism of M into $\mathbb{R} \times \mathbb{R} \times \dots$.