Inertial/Magnetic Sensors Based Orientation Tracking on the Group of Rigid Body Rotations with Application to Wearable Devices

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Abstract—In this work the problem of orientation tracking based on inertial/magnetic sensors is restated in geometric terms, in particular an intrinsic observer, i.e. an observer whose performance does not depend on a specific choice of coordinates, is derived on the Lie group of rigid body rotations $SO(3)$. Measurements of the gravitational and geomagnetic fields are used to estimate orientation errors. A coordinate-free control law is defined on the Lie algebra and fed back in terms of angular velocity that steers the observer towards the correct attitude. A proof of stability for the proposed estimator is provided which relies on the natural (bi-invariant) metric of $SO(3)$. The observer results stable for almost the whole configuration space. Presence of unstable equilibria as a limitation for global stability is also discussed. Based on the proposed intrinsic control law, a filter is designed which implements the observer. Simulations are presented that test the numerical implementation of the proposed observer.

I. INTRODUCTION

Motion tracking finds application in a broad set of fields, ranging from virtual reality to medicine, from film industry to biomedical research. As shown in [13] and references therein, although no “silver bullet” exists, motion tracking can count on a “respectable arsenal” of available technologies, each with its own advantages and disadvantages with respect to a given application.

At Campus Bio-Medico University, research is being carried out in the emerging field of Phenomics [9] with the aim of developing unobtrusive and ecological technologies which allow monitoring the behavior of infants and toddlers, especially during their first two years of life.

Behavior monitoring includes, for example, tracking a child’s posture, tracking the head direction (which mainly relates to the child’s attention), tracking position and/or orientation of toys while the child plays with them, etc…

At this moment, the most suitable technologies are being screened for this purpose. The selected ones will be deployed to be embedded in a child’s everyday environment, e.g. in toys and clothes. Special attention is paid to technologies which do not require costly equipment (e.g. photogrammetric systems) and/or a structured environment (e.g. motion analysis laboratories). The technology of interest should be able to work in clinical settings as well as at home and should be operated by minimally trained personnel, e.g. the child’s caregiver.

In this sense, orientation tracking based on inertial/magnetic sensors [7], [2], [14] represents a promising technology since, as shown in next sections, orientation of a rigid body can be measured solely relying upon gravitational and geomagnetic fields, which are present everywhere on earth, without the need of other sources of fields, i.e. sourceless.

Accelerometers and magnetometers are nowadays available in packages small enough to be worn or embedded into toys and can be used to track position/orientation in unstructured environments.

Theory of complementary and Kalman filters have traditionally been used to design attitude filters [7], [2], [14]. Besides describing experimental performance of the filters, no proof of stability of the proposed filters is provided and very little attention is paid to the geometric properties of the Lie group of rigid body rotations. Only few recent works [11], [8] address this topic as long as adaptive filters are concerned. As stated in [11], a geometric approach to the study of the dynamics and control of systems leads to results that are intrinsic, implying that performance will not depend of the choice of coordinates. Moreover if on a general manifold explicit control laws require coordinates, on Lie groups control laws can be stated on the Lie Algebra, leading to coordinate-free explicit expressions.

In what follows, the problem of attitude tracking via inertial/magnetic sensors is stated (for the first time to the authors’ knowledge) in geometric terms, then an intrinsic control law is proposed and finally the stability of the proposed observer on almost the whole configuration space is proved by exploiting the geometry of $SO(3)$ itself, presence of unstable equilibria as a limitation for global stability is also discussed. Based on the proposed proportional control law, a filter is designed which implements the observer. Simulations are presented that test the numerical implementation of the proposed observer.
II. **Left-invariant Dynamics on SO(3) and Proportional Control on the Lie algebra**

As shown in [1], [12], the natural configuration space for a rigid body is the Lie group \( SO(3) \), i.e. the configuration of a rigid body can always be represented by a rotation matrix \( R \), i.e. a matrix such that \( R^{-1} = R^T \) and \( \det R = +1 \)

Consider now the coordinate frames \( a, c \) where

- \( \mathbb{R}^3 \) : the space coordinate frame, or initial configuration frame.
- \( \mathbb{R}^3 \) : the body coordinate frame, which is attached to the body (can be thought of as defined by the sensors sensitive axis), initially coincident with the space frame.

An element \( R \in SO(3) \) can be thought of as a map from the body frame to the space frame, i.e. \( R : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \).

A trajectory of the rigid body is curve \( R(t) : \mathbb{R} \rightarrow SO(3) \). The velocity vector \( \dot{R} \) is tangent to the group \( SO(3) \) in \( R \) but, as shown in [1], [12], rather than considering \( \dot{R} \), two important quantities are worth to be considered:

- \( \dot{R} R^T \): representing the rigid body angular velocity relative to the space frame;
- \( R^T \dot{R} \): representing the rigid body angular velocity relative to the body frame.

These are both elements of the Lie algebra \( so(3) \), i.e. the tangent space to the group \( SO(3) \) at the identity \( I_3 \).

Elements of the Lie algebra are represented by skew-symmetric matrices. Systems on Lie groups described in terms of body (space) coordinates are called left-invariant (right-invariant).

**Left-invariance:** let \( R_1(t) \) be a trajectory of a rigid body relative to a space frame \( \mathbb{R}^3_S \). Consider a change of space frame \( G : \mathbb{R}^3_S \rightarrow \mathbb{R}^3_S \), now \( R_2(t) = G R_1(t) \) represents the same trajectory but with respect to the new space frame. It is straightforward to verify that \( R_2^T \dot{R}_2 = R_1^T \dot{R}_1 \), i.e. the angular velocity relative to the body frame \( R^T \dot{R} \) does not depend on the choice of space frame.

**Right-invariance:** similarly, it can be shown that the angular velocity of a rigid body relative to a space frame \( R R^T \) does not depend on the choice of coordinate frame attached to the body.

In the case of \( SO(3) \), there exists [12] an isomorphism of vector spaces \( \gamma : so(3) \rightarrow \mathbb{R}^3 \) that allows writing \( so(3) \approx \mathbb{R}^3 \).

For a given vector \( a = [a_1, a_2, a_3] \in \mathbb{R}^3 \), write:

\[
\gamma : a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \approx \bar{a} \quad (1)
\]

The Lie algebra is equipped with an operator, the Lie bracket \([\cdot, \cdot]\) which is defined by the matrix commutator:

\[
\lbrack \bar{a}, \bar{c} \rbrack = \bar{a} \bar{c} - \bar{c} \bar{a} = a \times c
\]

where \( a, c \in \mathbb{R}^3 \), \( \bar{a}, \bar{c} \in so(3) \) and \( \times \) is cross product in \( \mathbb{R}^3 \).

**A. Left-invariant tracking on SO(3)**

Motion control of left-invariant systems evolving on Lie groups is widely explored in literature, see [10], [4] and references therein. Recently such approaches have been extended to observers as well, e.g. [8], [11].

**Statement of the problem:** given a rigid body initially at rest in a configuration \( R(0) = I_3 \in SO(3) \) and reaching, after some time, a final steady configuration \( R \in SO(3) \), determine a control law \( \Omega^* \) (angular velocity relative to the body) for the observer:

\[
\begin{cases}
R(t) \dot{R}^* = \Omega^* \\
R(t)(0) = R(0) = I_3 \in SO(3)
\end{cases}
\]

such that \( \Omega^* \) is solely based on gravitational and geomagnetic fields sensing and with the property that \( R^*(t) \) converges asymptotically to \( R \), i.e.:

\[
\lim_{t \rightarrow \infty} R^*(t) = R
\]

**Remark:** the choice of referring quantities to the body frame (left-invariance) is a natural one since sensors provide measurements with respect to the body coordinates. This will result clear in the choice of a control law.

As in every feedback control problem, first an error function will be derived (in this case based on sensors readings) and then a correction will be fed back to the plant as a velocity correction.

**B. Proportional control on the Lie algebra so(3)**

Gravitational and geomagnetic fields can be used to estimate rotations of a rigid body relative to an earth-fixed coordinate frame, see [6] and references therein. Accelerometers and magnetometers can be used to derive gravity and magnetic vectors relative to the body coordinate \( \mathbb{R}^3_B \).

Let \( g_0, b_0 \in \mathbb{R}^3_B \) be the readings of the sensor at time \( t = 0 \), i.e. when the body frame and the space frame coincide \( (R(0) = I_3) \).

At a generic time \( t \), after the final configuration \( R \) is reached, the readings would be \( b = R^T b_0 \) and \( g = R^T g_0 \), since the gravity and magnetic vectors are fixed with respect to the space frame but vary in the body frame. Such vectors carry an information relative to the configuration \( R \) to be estimated. If \( R^* \) is the current estimate (at time \( t \)) of the configuration \( R \), then the expected readings out of the sensors would be \( b^* = R^T b_0 \) and \( g^* = R^T g_0 \).

Any misalignment between \( b \) and \( b^* \) or between \( g \) and \( g^* \) is a “symptom” of a difference between \( R \) and its estimate \( R^* \).

We are looking for a correction in terms of velocity (relative to the body) to be added to the observer in order to re-gain alignment.

The “shortest” way (see discussion on metric properties, below) to regain alignment between \( b \) and \( b^* \) is by imposing an angular velocity relative to the body frame proportional to \( b \times b^* \). Clearly, when \( b \) and \( b^* \) are aligned, their vector product is zero, i.e. no correction is needed. If an angle, say \( \theta_b \), exists between the two vectors, the correction, proportional to \( \sin \theta_b \), is an infinitesimal generator [12] of a rotation along the axis perpendicular to both \( b \) and \( b^* \).

Similarly, to realign \( g \) and \( g^* \) a velocity proportional to \( g \times g^* \) should be imposed.
Proposed feedback: the following proportional control, based on sensors read-outs, for the observer is proposed:

\[ \Omega^* = k_b (b \times b^*) + k_g (g \times g^*) \quad (2) \]

which translates into a final dynamic system for the observer:

\[ R^* \dot{R}^* = k_b (b \times b^*) + k_g (g \times g^*) \quad (3) \]

The observer can be implemented via the filter in Fig.1 where \( b \) and \( g \) are the current sensors read-outs while \( b_0 \) and \( g_0 \) are the initial sensors read-out, i.e. \( b_0 = b(0) \) and \( g_0 = g(0) \). Not shown in the figure is the initial condition for the integrator block, i.e. \( R^*(0) = I_3 \).

Next section will analyze convergence properties of the proposed observer but, before that, metric properties of \( SO(3) \) will be briefly reviewed.

C. Metric properties of \( SO(3) \)

In what follows necessary background and geometric tools are reviewed in order to define a norm (a distance) on \( SO(3) \). This will be used later to prove convergence of the proposed feedback.

On a general manifold \( M \), a positive definite quadratic form \( \langle \xi_1, \xi_2 \rangle_{T_x M} \) defined on any tangent space \( T_x M \not\equiv \xi_1, \xi_2 \) (the space tangent to \( M \) in \( x \in M \)) is called a Riemannian metric [1]. It is the equivalent of the scalar product in \( \mathbb{R}^n \), can be used to measure the distance between different points of a manifold, in mechanics a metric is tightly linked to the definition of kinetic energy [5]. A metric is extra structure, does not come with the manifold. Many different metrics, i.e. many different distance measures, can be defined on the same manifold [1].

Lie groups are, by definition, manifolds and therefore are entitled to posses metric properties. Lie groups, in particular \( SO(3) \), are structured in such a way that some metrics naturally\(^1\) arise. A left-invariant metrics does not depend on the choice of the space frame, i.e. it only needs to be defined on the Lie algebra and then it can be left-translated to the tangent space at any other group element:

\[ \langle R \hat{a}, R \hat{c} \rangle_{SO(3)} = \langle \hat{a}, \hat{c} \rangle_{so(3)} \]

where \( R \in SO(3) \) and \( \hat{a}, \hat{c} \in so(3) \).

Still, there many choices for a metric in the Lie algebra, as many as there are positive definite matrices \( P \):

\[ \langle \hat{a}, \hat{c} \rangle_{so(3)} \overset{\Delta}{=} a^T P c \]

where \( a, c \in \mathbb{R}^3 \) correspond to \( \hat{a}, \hat{c} \in so(3) \) as in Eq.(1). However, there only exists one choice (up to a coefficient, [12], [4], [5], [1]) when the metrics needs to be also right-invariant:

\[ \langle \hat{a}, \hat{c} \rangle_{so(3)} \overset{\Delta}{=} a^T I_3 c = \langle a, c \rangle \quad (4) \]

where \( \langle a, c \rangle \) is the dot product in \( \mathbb{R}^3 \).

Two main results provided in [4] are:

- the existence of a natural norm\(^2\) on \( SO(3) \)

\[ \| R \|_{SO(3)} = \langle \hat{\phi}_R, \hat{\phi}_R \rangle_{so(3)}^{1/2} = \| \phi_R \| \]

- time derivative of such a norm:

\[ \frac{1}{2} \frac{d}{dt} \| R(t) \|_{SO(3)} = \langle \hat{\phi}_R, R^T \dot{R} >_{so(3)} \]

where \( \hat{\phi}_R \in so(3) \), also referred to as log \( R \), is defined as the angular velocity that takes the rigid body from \( I_3 \) to \( R \in SO(3) \) in one time unit, see [12] for details on the logarithmic map:

\[ \hat{\phi}_R = \log R = \frac{\theta}{2 \sin \frac{\theta}{2}} (R - R^T) \quad (5) \]

where, for \( \text{trace}(R) \neq 1 \), \( \theta \) satisfies \( 1 + 2 \cos \theta = \text{trace}(R) \) and \( \| \phi_R \|^2 = \theta^2 \), and the Rodrigues’ formula:

\[ R = \exp(\hat{\phi}_R) = I_3 + \alpha_R \hat{\phi}_R + \beta_R \hat{\phi}_R \quad (6) \]

where \( \alpha_R = 1 - \| \phi_R \|^{-1} \sin \| \phi_R \| \) and \( \beta_R = (1 - \cos \| \phi_R \|) / \| \phi_R \|^2 \).

Remark: after a metric on \( SO(3) \) is defined, it is possible to see how the proposed feedback in Eq.(2) represents the “shortest” way to realign the fields \( b^* \) and \( g^* \) respectively along \( b \) and \( g \). A metric allows in fact defining geodesics, i.e. trajectories along which the shortest paths between two points on a manifold run [11], [5].

III. CONVERGENCE PROPERTIES

In this section, as in [4], a candidate Lyapunov function is determined based on the metric properties of \( SO(3) \) and is used to prove convergence of the observer.

Theorem: Given the observer equations:

\[ R^* \dot{R}^* = k_b (b \times b^*) + k_g (g \times g^*) \quad (7) \]

\(^1\)Natural means that it does not depend on a particular choice of coordinates.

\(^2\)Which measures the distance between \( R \) and the identity \( I_3 \).
where $b = R^T b_0$, $b^* = R^{*T} b_0$, $g = R^T g_0$, $g^* = R^{*T} g_0$
and where $b_0$ and $g_0$ are the initial (time $t = 0$) measurements of the geomagnetic and gravitational fields, i.e. when $R^* = R(0) = I_3$, the observer will track any final steady configuration $R$ such that $\text{trace}(R) \neq -1$, i.e.:
$$\lim_{t \to \infty} R^*(t) = R$$

**Proof:** Define first the error $E \in SO(3)$ as in [4]:
$$E \triangleq R^T R^*$$
and compute its derivative (considering that $\dot{R} = 0$ since $R$ is a steady final configuration):
$$\dot{E} = R^T \dot{R}^*$$
Rewrite now Eq.(7) in terms of $E$:
$$E^T \dot{E} = R^T R^* = k_b \hat{\Omega}_b + k_g \hat{\Omega}_g$$
where $\hat{O}_b$ and $\hat{O}_g$ are defined as follows:
$$\Omega_b = b \times b^* = (R^T b_0) \times b^* = (R^T R^* R^T b_0) \times b^* = (E R^* b_0) \times b^* = (E b^*) \times b^*$$
Via the Rodrigues’ formula in Eq.(6), $E$ can be expressed in terms of $\log E$ (also denoted $\hat{\phi}_E$) in the Lie algebra $so(3)$:
$$E = \exp(\hat{\phi}_E) = I_3 + \alpha_E \hat{\phi}_E + \beta_E \hat{\phi}_E^2$$

so we can write:
$$\Omega_b = \left((I_3 + \alpha_E \hat{\phi}_E + \beta_E \hat{\phi}_E^2) b^* \right) \times b^* = b^* \times b^* + \alpha_E (\hat{\phi}_E b^*) \times b^* + \beta_E (\hat{\phi}_E^2 b^*) \times b^* = \alpha_E (\hat{\phi}_E b^*) \times b^* + \beta_E (\hat{\phi}_E^2 b^*) \times b^*$$

Since $b^* \times b^* = 0 \forall b^* \in \mathbb{R}^3$, and similarly:
$$\Omega_g = \alpha_E (\hat{\phi}_E g^*) \times g^* + \beta_E (\hat{\phi}_E^2 g^*) \times g^*$$
Consider now the following positive definite candidate Lyapunov function:
$$W(E) = \frac{1}{2} \| E \|^2_{SO(3)} = \frac{1}{2} \| \hat{\phi}_E \|^2$$
where $\hat{\phi}_E = \log E \in so(3)$ is defined as in Eq.(5):
$$\log E = \hat{\phi}_E = \frac{\| \hat{\phi}_E \|}{2 \sin \| \hat{\phi}_E \|} (E - E^T)$$

**Note:** the condition $\text{trace}(E(0)) = \text{trace}(R) \neq -1$ implies that, at least at $t = 0$, $\| \hat{\phi}_E \| < \pi$.
In order to prove convergence, the sign of its time derivative should be studied:
$$\dot{W}(E) = \frac{d}{dt} \| E \|^2_{SO(3)} = \langle \log E, E^T \dot{E} \rangle_{so(3)}$$
where $1 + 2 \cos \| \hat{\phi}_E \| = \text{trace}(E)$, for $\text{trace}(E) \neq 1$, while the $E^T \dot{E}$ term is given by Eq.(8).

As in Eq.(4), the scalar product in $so(3)$ can be evaluated via the dot product in $\mathbb{R}^3$:
$$\langle \hat{\phi}_E, k_b \hat{\Omega}_b + k_g \hat{\Omega}_g \rangle_{so(3)} = \langle \hat{\phi}_E, k_b \hat{\Omega}_b + k_g \hat{\Omega}_g \rangle = k_b \langle \hat{\phi}_E, \hat{\Omega}_b \rangle + k_g \langle \hat{\phi}_E, \hat{\Omega}_g \rangle$$

Consider $\langle \hat{\phi}_E, \hat{\Omega}_b \rangle$ first:
$$\langle \hat{\phi}_E, \hat{\Omega}_b \rangle = \langle \hat{\phi}_E, \alpha_E (\hat{\phi}_E b^*) \times b^* + \beta_E (\hat{\phi}_E^2 b^*) \times b^* \rangle = \alpha_E \langle \hat{\phi}_E, b^* \rangle \times b^* + \beta_E \langle \hat{\phi}_E, \hat{\phi}_E^2 b^* \rangle \times b^*$$

Before proceeding, it can be verified\(^3\) that the following identities hold $\forall a, b \in \mathbb{R}^3$ and $\hat{a} \in so(3)$:
$$\langle \hat{a} b \times b \rangle = < a, b > b - \| b \|^2 a$$
$$< a, (\hat{a}^2 b) \times b > = 0$$
Which directly translates into:
$$\begin{cases}
\langle \hat{\phi}_E, \hat{\Omega}_b \rangle = \alpha_E \langle \hat{\phi}_E, b^* \rangle^2 - \| b^* \|^2 \| \hat{\phi}_E \|^2 \\
\langle \hat{\phi}_E, \hat{\Omega}_g \rangle = \alpha_E \langle \hat{\phi}_E, g^* \rangle^2 - \| g^* \|^2 \| \hat{\phi}_E \|^2 
\end{cases}$$
Recalling that the dot product in $\mathbb{R}^3$ can be written:
$$< a, b >= ||a|| ||b|| \cos \theta$$
where $\theta$ is the angle between vectors $a, b \in \mathbb{R}^3$, then:
$$\begin{cases}
\langle \hat{\phi}_E, \hat{\Omega}_b \rangle = \alpha_E \langle \hat{\phi}_E, b^* \rangle^2 - \| b^* \|^2 || \hat{\phi}_E \| \cos \theta \\
\langle \hat{\phi}_E, \hat{\Omega}_g \rangle = \alpha_E \langle \hat{\phi}_E, g^* \rangle^2 - \| g^* \|^2 || \hat{\phi}_E \| \cos \theta 
\end{cases}$$
where $\theta_b$ and $\theta_g$ are the angles between $\phi_E$ and, respectively, $b^*$ and $g^*$.

Recalling also that $\alpha_E = || \hat{\phi}_E ||^{-1} \sin || \hat{\phi}_E ||$, the time derivative of the Lyapunov function can thus be written as:
$$\dot{W}(E) = -|| \hat{\phi}_E || \sin || \hat{\phi}_E || (k_b \| b^* \|^2 (1 - \cos^2 \theta_b) + k_g \| g^* \|^2 (1 - \cos^2 \theta_g))$$

the term $(1 - \cos^2 \theta_b)$ (respectively $(1 - \cos^2 \theta_g)$) is always non-negative, it is zero only when $\cos \theta_b = \pm 1$ (respectively $\cos \theta_g = \pm 1$), this condition only occurs when the vector $\phi_E$ is parallel to $b^*$ (respectively $g^*$). Since $b^*$ and $g^*$ are the measurements of geomagnetic and gravitational fields they are independent vectors, in particular never parallel. For this reason an arbitrary vector $\phi_E$ can never be aligned with $b^*$ and $g^*$ at the same time, i.e. $(1 - \cos^2 \theta_b)$ and $(1 - \cos^2 \theta_g)$ can never be zero at the same time.
In fact, it can be shown that $\forall k_b, k_g > 0 \in \mathbb{R}$, $\exists \lambda > 0 \in \mathbb{R}$ such that:
$$(k_b \| b^* \|^2 (1 - \cos^2 \theta_b) + k_g \| g^* \|^2 (1 - \cos^2 \theta_g)) \geq \lambda > 0$$
Recalling the definition of $W(E)$ in Eq.(9), we can now write:
$$\dot{W}(E) \leq -\lambda \sqrt{2W(E)} \sin \sqrt{2W(E)}$$
or, with a simpler notation:
$$\dot{W} \leq -\lambda \sqrt{2W} \sin \sqrt{2W}$$
\(^3\)Note that $\hat{a} b = a \times b$ and that $\hat{a}^2 b = a \times (a \times b)$.
Consider a rotation $R_\pi$ of $\pi$ radians about an axis perpendicular to plane containing both the gravity vector $g_0$ and the geomagnetic vector $b_0$. It’s straightforward verifying that $R_\pi g_0 = -g_0$ and that $R_\pi b_0 = -b_0$, i.e. the correction terms for the observer are zero, i.e. $R_\pi$ is an equilibrium (of course an unstable one). There also exist other unstable equilibria among the $\pi$-rotations, some of which depend also on the values of $k_g$ and $k_b$.

**Remark:** The presence of unstable equilibria among the $\pi$-rotations excludes the possibility of a better choice for the Lyapunov function. In practice, unstable equilibria do not represent an issue due to the presence of noise.

### IV. Validation via Numerical Simulation

Convergence properties of the proposed observer were tested via numerical simulations. First of all the observer was implemented in Simulink/MATLAB environment according to the block diagram in Fig.1. The only difference is that vectors $b$ and $g$ where first normalized. The reason for doing so is that by normalizing each vector, the values for $k_g$ and $k_b$ no longer depend upon unit of measurement ($m/s^2$ or Tesla) and, more important, sensors read-outs (expressed in Volts) can be directly used as inputs for the observer.

A first simulation in Fig.3 shows the performance of the filter while tracking three different steady state configurations although starting from the same initial configuration $I_3$.

Another simulation is presented in Fig.4 where, starting from $I_3$, the same final configuration (the same as the right
sequence in the previous simulation) is reached but with different speeds. The convergence speed can be regulated by properly choosing $k_g$ and $k_b$. In Fig. (4), the slow, medium, and fast sequences respectively correspond to $k_g = k_b = 0.6$, $k_g = k_b = 1$, and $k_g = k_b = 2$.

V. Conclusion

In this work, for the first time to the authors’ knowledge, the problem of attitude tracking via inertial/magnetic sensors was stated in the group of rigid body rotations $SO(3)$. Such an approach fully exploits the left-invariant character of the problem and the fact that sensors provide information relative to the body frame. Misalignment between sensor readings and estimated direction of gravitational and geomagnetic fields thus defines an error measure which naturally lives on the Lie algebra and that, as such, can be directly used as proportional feedback in terms of angular velocity relative to the observer.

Stability of the proposed observer is proved for almost the whole configuration space $SO(3)$. The presence of unstable equilibria as a limit to global stability is also discussed. An implementation of the observer is proposed and numerically validated via simulations. As future work, convergence properties of the proposed observer will be analyzed in presence of noisy data, disturbances such as non-inertial accelerations and geomagnetic field distortion.

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