Abstract

In this paper, an efficient charge recovery method for driving piezoelectric actuators with low frequency square waves in low power applications such as mobile microrobots is investigated. Efficiency issues related to periodic mechanical work of the actuators and the relationship among the driving electronics efficiency, the piezoelectric coupling factor, and the actuator energy transmission coefficient are discussed. The proposed charge recovery method exploiting the energy transfer between an inductor and a general capacitive load is compared with existing techniques which lead to inherent inefficiencies. A charge recovery method is then applied to piezoelectric actuators, especially to bimorph ones. Unitary efficiency can be theoretically obtained for purely capacitive loads while intrinsic losses such as hysteresis necessarily lower the efficiency. In order to show the validity of the method, a prototype driving electronics consisting of an extended H-bridge is constructed and tested by experiments and simulations. Preliminary results show that 75% of charge (i.e. more than 56% of energy) can be recovered for bending actuators such as bimorphs without any component optimization at low fields.
I. Introduction

Piezoelectric actuators are widely used in smart structure applications due to their high bandwidth, high output force/torque, compact size, and high power density properties. As a new emerging application area, they have been utilized in mobile microrobotic applications such as micromechanical flying insects (MFI) [1] and small cockroach robots [2]. For these robots, the overall size and weight are limited for biomimetic locomotion, e.g. flying, walking, or jumping. Thus, there are critical limitations on the actuator and its driving electronics. In this paper, MFI is the target application, and a compact driving electronics for piezoelectric bending actuators is the goal. The specifications of the driving electronics are quasi-square wave driving at around 150 Hz resonant frequency, weight around 20 – 30 mg, mechanical power output of 10 mW, and voltage source of 3 – 10 V using a thin-film lithium battery or microfabricated solar cells [3].

Thick film piezoelectric actuators require high input voltages. For high voltage driving, switching electronics or analog amplifiers are utilized. Since the latter would lead to excessive dissipation [4], the former is preferred. For the MFI project, boost or flyback DC-DC converters [6] will be employed to step up the voltage provided by a battery or a solar cell. After the step-up stage, a half or full bridge switching stage converts the high voltage into a square wave across the load. Due to the capacitive nature of the load, a significant amount of power is lost at any switching cycle if standard converters are employed. Therefore special care has to be taken to analyze the relationship between overall efficiency in squarewave driving and figures of merit for energy conversion of piezoelectric actuators. When piezoelectric materials are used, conversion from electrical energy into mechanical energy is primarily characterized by the coupling factor [5], defined as the ratio of the mechanical energy stored in the volume of the piezoelectric material over the input electrical energy. The coupling factor is well known to be much smaller than unity, i.e. only a relatively small fraction of the input electrical energy can be converted into stored mechanical energy. Furthermore, depending upon the geometry of the actuator, the way it is mechanically coupled to the load and the load itself (e.g. a constant force such as gravity or a non-constant one such as a spring), only a fraction of the mechanical energy stored in the volume of the piezoelectric material can be transferred to the mechanical load. For this purpose the energy transmission coefficient can be introduced which is defined as the ratio of output mechanical energy over input electrical energy [2],[7]. Despite the fact that only a small fraction of the input electrical energy, stored in the volume of the piezoelectric material, can be transformed into mechanical work done on the load, energy transfer can still be characterized by high efficiency.
Efficiency of a system is defined [8] as the ratio of the output energy over the consumed electrical energy. Although a large amount of electrical energy may be required to be stored at certain time, only part of it will be transformed into mechanical work. The remainder is unused energy that can be returned to the power source [8]. As also shown in [4], part of the stored electrical energy is in fact recoverable.

In [4], recovered energy is temporarily stored in external capacitors and returned in successive cycles. As shown later, such a method has intrinsic disadvantages since charging and discharging a capacitor by directly connecting it to other capacitors and/or power supplies necessarily leads to 50% energy loss even if ideal (i.e. lossless) actuators were considered. The aim of this work is to increase the efficiency when subjecting a general capacitive actuator to energy cycling by means of square wave driving. For this purpose, an external inductor is used which allows to achieve unitary efficiency, at least theoretically, for a purely capacitive load. Next, the nonideal case, i.e. real piezoelectric actuators with inherent losses such as hysteresis, are analyzed.

To implement a charge recovery H-bridge, an inductor for temporarily storing the energy and two diodes for self-timed switching are added to a standard H-bridge (a.k.a. full bridge in [6]). A prototype driving stage with no component optimization is implemented and experimental results are shown.

II. QUASI-SQUARE WAVE DRIVING METHODS

For high efficiency piezoelectric actuation, switching output stages are preferred [4] where the losses only occur at the switching time. For this case, only two output states are available, and therefore pulse width modulation (PWM) techniques are often used [4]. In PWM systems, dissipation linearly increases with the frequency.

Switching output stages give rise to a quasi-square wave voltage across the load and can be referred to as bipolar when the voltage is switched between $\pm V_0$ and unipolar when voltage is switched between $V_0$ and 0. Piezoelectric actuators can be represented [9] as comprising a capacitor $C$ and mechanical impedance $Z_m$ typically characterized by at least one dominant resonance frequency $\omega_m$, i.e. an $LRC$ electrical equivalent circuit [5]. In order to excite resonance in the actuator-load system, actuators are driven by square waves at frequency $\omega_m$, thus during most of the time voltage is held constant performing no work on the capacitor $C$ while transferring power to the mechanical load, only at switching time will power be dissipated by the capacitor while mechanical impedance will have small or no effect due to its slow response. Although nonzero, switching time will always be negligible compared to the wave’s period, corresponding to the period of mechanical resonance.
Charge recovery methods focus on reducing losses occurring at switching time, when piezoelectric actuators are mainly characterized by a (lossy) capacitive behavior, and aim to temporarily store the charge of a pre-charged capacitor before discharging it or before reversing its polarity.

A. *Intrinsic Limits of Standard Charge Recovery Methods*

Power dissipation due to periodic charging and discharging of capacitive loads has been an issue also for digital electronics technology employing CMOS logic [10],[11]. As a solution, the operating voltage level is being reduced in digital circuits. However, such a solution is not feasible for piezoelectric actuator applications where usually high stored energy is necessary.

Considering a purely capacitive load, two well known facts are to be taken into account:

- Charging up an initially discharged capacitor $C$ by means of a constant voltage source $V$ and a (resistive) switch requires a total energy $E = QV = CV^2$ that is exactly twice the final energy stored in the capacitor itself independently of switch resistance. The final energy stored in the capacitor is thus $\frac{1}{2}CV^2$ and the same amount has been dissipated across the resistive switch.

- When connecting a charged capacitor $C_1$ with a discharged one $C_2$ by means of a resistive switch, the final energy will be less than the initial one. When $C_1 = C_2 = C$, which is the case of interest\footnote{Energy transfer is bidirectional, in case of different capacitors transfer efficiency in one direction (say $\eta_1$) is advantaged at the expense of the other direction ($\eta_2$). Final efficiency will be $\eta = \eta_1 \times \eta_2$ and it is straightforward to see that only a symmetric situation (i.e. $C_1 = C_2$) leads to maximizing overall efficiency $\eta$.}, the final voltage would be half of the initial one, thus only 50% of charge (i.e. 25% of energy) would be recovered.

As the first point clearly suggests, directly switching a capacitive load between positive and negative constant voltage sources is *inefficient in principle* and before connecting it to a negative power supply, its charge should be stored and returned later.

[4] attempts to store such an energy in another capacitor by simply connecting it to the actuator, but the second point underlines unnecessary losses arising when two capacitors are directly connected to one another.

Although reversing polarity of a pre-charged capacitor or transferring its charge to a similar discharged one may seem different problems they in fact rely upon the same principle.

The purpose of this work is to focus on a *natural* principle, theoretically lossless, which aims to extract the whole charge from a pre-charged capacitor and use it to immediately return it with reversed polarity (bipolar fashion) or to store it in another capacitor and return it in the next cycle (unipolar fashion). Resonance, between an inductor and a capacitor, is in principle an efficient and simple way to reverse...
the polarity of a capacitive load. Energy is initially stored in the inductor and then returned to the capacitor with reversed voltage polarity. Theoretically, with lossless components, unitary efficiency can be achieved. In the next section it will be shown how the natural resonance principle occurring in a $LC$ circuit can be exploited to recover charge.

**B. General Charge Recovery Principle for Capacitive Loads**

First, the issue of reversing the polarity of a pre-charged capacitor is described. For this purpose an inductor $L$ can be deployed to resonate with the pre-charged capacitor $C$ at frequency $\omega_0 = 1/\sqrt{LC}$. Referring to Fig. (1)a, a switch $S$ is used to start the resonance, and a diode $D$ is used to automatically interrupt it exactly after half of a cycle. When no losses are considered, basic circuit theory shows that after half of cycle, the voltage across the capacitor will be reversed and that this voltage will be kept constant independently of the status of the switch.

Linear losses will be considered later. Here it is worth noticing that the presence of the diode will avoid the need for accurate timing. In fact, without any diode, the switch should be turned off at exactly half of the period since a failure in doing so would lead to incomplete recovery of the energy stored in the capacitor. Now that the working principle has been outlined, its application to quasi-square wave driving stages will be discussed.

**C. Application of Charge Recovery Principle to Bipolar Square Wave Switching Stages**

When linear losses are considered, the oscillation occurring in the circuit in Fig. (1)a will simply exponentially decay to zero. In particular, after half of a cycle the voltage across the capacitor will be $-\eta V_0$, where $V_0$ is the original voltage across the capacitor and $0 < \eta < 1$ is the charge recovery efficiency ($\eta = e^{-\pi Q}$, where $Q$ is the quality factor of the oscillating circuit, including losses). Recovery is thus not complete and a power supply has to be employed to force the capacitor (load) to a final voltage $-V_0$. It is worth noticing how, so far, in order to switch from $V_0$ to $-\eta V_0$ no work has been done by the power supply at all. The closer $\eta$ is to unity the less electrical work is necessary from the power supply.

Fig. (1) shows how polarity of a positively (a) and negatively (b) pre-charged capacitor is reversed, the two circuits only differ in orientation of the diodes. The two circuits can merge (Fig. (1)c) to provide both transitions and the same inductor can be shared leading to reduction of size and weight, which is of primary importance in applications such as [1]. Two switches ($S4$ and $S1$) are added to force the capacitor to a final voltage of $V_0$ or $-V_0$. 
D. Application of Charge Recovery Principle to Unipolar Square Wave Switching Stages

Although bipolar and unipolar square waves are different ways of driving a capacitive load, the charge recovery principle can be applied to both.

As shown in Fig. (2)a, an inductor can still be used to connect a charged capacitor to a discharged one. It can easily be verified that if their capacitances are similar, in an ideal lossless situation, charge can be completely moved from one to another as schematically depicted in Fig. (2)b.

Notice that it is still a resonating LC circuit where two capacitors \( C \) are in series, equivalent to a single capacitor of value \( C/2 \).

Considering left and right capacitors \( C_L \) and \( C_R \), charge can be moved from left to right or vice versa with the circuit shown in Fig. (3), where similarly to the bipolar case, two circuits are merged and extra switches are added to achieve full voltages \( V_0 \) and 0 after partial (\( \eta \)) transfer of the charge from one to another.

Effectiveness of the circuit depicted in Fig. (3) is mainly based on symmetry, thus it is necessary to have \( C_L = C_R \). Given a load characterized mainly by its parasitic capacitance \( C_0 \), a pure capacitor of value \( C_0 \) can be used to match it in the aforementioned circuit. Application of such a circuit to unimorph and bimorph actuators will be discussed below.

E. Implementation Details and Linear Losses

Prototypes for both circuits in Fig. (1)c and Fig. (3) were realized without any attempt at component optimization but with the only aim to prove feasibility of the charge recovery principle. All switches have been implemented by NMOS (ZVN4424) or PMOS (ZVP4424) transistors. In Fig. (1)c, PMOS have been used for \( S_4 \) and \( S_2 \) with NMOS for \( S_1 \) and \( S_3 \). Thus, every transistor has the source pin connected to ground, \(+V_0\), or \(-V_0\), and gate voltage could easily be referred to one of these fixed voltages.

In Fig. (3), PMOS have been used for \( S_{HR} \) and \( S_{HL} \) and NMOS for all other switches. Special care must be taken with \( S_1 \) and \( S_2 \) since their source pin cannot be connected to ground or to \( V_0 \), therefore floating circuitry such as batteries and optocouplers has been used to drive their gate.

Referring to Fig. (1)a, when diode is forward biased, each device was be assumed to be working within the linear range, a condition that was verified from datasheets for the prototyped circuit. Since all components are in series, their resistance can be summed up. It is convenient to consider the quality factor \( Q \) of the oscillating circuit instead of its resistance. Although \( Q = Q(\omega) \) is in general frequency-
varying, only $Q(\omega_0)$ is relevant, where $\omega_0 = 1/\sqrt{LC}$. The inductor should be chosen in order to have minimum losses at the desired resonating frequency.

Previous considerations only hold if devices work within the linear range; in order to verify it, an upper bound of peak current is needed. Considering no losses (overestimation of current), the energy initially stored in the capacitor $E_C = \frac{1}{2}CV_0^2$ will completely be transferred into the inductor at one fourth of a resonating period, i.e. the moment of peak current $I_0$, where $E_L = \frac{1}{2}LI_0^2$. By conservation of energy (no losses) $E_C = E_L$ which easily leads to:

$$I_0 = V_0\sqrt{\frac{C}{L}}$$  \hspace{1cm} (1)

Such an upper bound for the peak current flowing during switching allows one to verify whether every device works in the linear range or not.

Fig. (4) shows normalized voltages across capacitors $C_R = 40 \, nF$ and $C_L = 40 \, nF$ for the unipolar circuit of Fig. (3) soon after switching. Although no attempt of optimization has been made, energy efficiencies higher than 80% have been achieved. In this particular case an inductor $L = 220 \, \mu H$ was used and the resulting half period is:

$$\frac{T}{2} = \pi \sqrt{L \frac{C_R C_L}{C_R + C_L}} \approx 6.5 \, \mu sec$$  \hspace{1cm} (2)

The switching time is about $6\mu sec$ while the whole square wave period is about $6\,ms$, corresponding to a real insect wing beat frequency of about $150 \, Hz$ [1]. At this scale, the switching time is negligible and thus it can be considered instantaneous, which is consistent with the quasi-square wave assumption.

### III. Application to Piezoelectric Actuators

In this section a circuit prototype implementing charge recovery is applied to piezoelectric actuators. The circuit shown in Fig. (3) can be applied to similar piezoelectric actuators (unimorph, stack actuators or any kind of actuator presenting two electrodes) by simply replacing the capacitors $C_L$ and $C_R$ with the actuators themselves.

When actuators present more electrodes, such as bimorph ones, each piezoelectric layer can replace one capacitor in Fig. (3).

A bimorph actuator can, in principle, be considered as two unimorphs bonded together back to back. In order to avoid depoling, a single unimorph can only be driven in unipolar fashion and its resulting bending will therefore be unidirectional as well. When the charge recovery circuit drives a bimorph, only a layer at a time is driven while the other is short-circuited (zero voltage is imposed across its
electrodes), behaving thus as an elastic layer. During the second half of a square wave cycle the situation is reversed, resulting in bidirectional bending.

A. General Linear Model For Piezoelectric Actuators And Its Applicability To Charge Recovery Principle

Detailed models of piezoelectric actuators have been developed during last few decades. [5] is indeed a good source for linear description of piezoelectric materials such as PZT. Whether bonded to external mechanical structures (such as cantilevers, plates, shells), clamped, or freely oscillating, piezoelectric actuators can be described as a vibrating mechanical structure coupled with an electric field acting as a generalized mechanical force [5]. Such coupling is bidirectional so that piezoelectric actuators can be modelled by linear multi-port networks, at least within certain operating ranges. One (or even more, as for bimorphs) of these ports represents an actuator’s electrodes and linear network theorems can be applied in order to reduce the whole (linear) model to an electrical two-port whose impedance mirrors mechanical properties.

When dealing with a piezoelectric actuator, instead of considering its impedance $Z(j\omega)$ (seen from the electrical port) or equivalently its admittance $Y(j\omega)$, it is often useful to describe it by means of its generalized capacitance $C(j\omega) = Y(j\omega)/j\omega$ as in [12]. A piezoelectric actuator will in fact behave as a (lossy) capacitance at almost any frequency except for sharp ranges around mode frequencies, as shown in the schematic diagram in Fig. (5)a. An analytical description for the bimorph case can be found in [12] whose results, together with [5], can be generalized into the diagram in Fig. (5)a.

In [5] it is also shown how to approximate the frequency-dependent behavior of Fig. (5)a with a lumped parameter model as shown in Fig. (5)b. Each mechanical mode is represented by an $RLC$ circuit. Resistors $R_i$ represent linear losses; in the general case, they are frequency dependent as well. A main issue is whether resistors $R_i$ can take into account any kind of losses occurring in a piezoelectric actuator, at least in the case of interest, i.e. when driven by a charge recovering circuit. Piezoelectric actuators are well known to be affected by hysteresis, which is typically a non-linear type of loss. Nonlinear models are presented in [9] for hysteresis, which is shown to occur only in the dielectric domain, i.e. it only affects electric field $E$ and dielectric displacement $D$ (corresponding respectively to global variables voltage $V$ and charge $Q$ at actuator’s electrodes).

In [9], hysteresis is also shown to be rate independent, i.e. applying a periodic voltage to a piezoelectric actuator and measuring the current flowing through it leads to a voltage vs. charge plot which does not depend upon frequency of input voltage nor upon the voltage waveform (sinusoidal, saw-tooth or
other), the area surrounded by the hysteresis curve is equivalent to the energy lost in a cycle. In Fig. (5)b, dielectric domain is described by the $R_0C_0$ branch, while any other $R_iL_iC_i$ branch simply reflects mechanical behaviors. It is obviously impossible to perfectly describe hysteresis with linear resistors (otherwise hysteresis would be a linear phenomenon). Nevertheless, the charge recovering circuit is expected to drive the piezoelectric actuators with currents and voltages resembling (half a period of) sinusoidal waveforms, similar to the pure capacitor case (Fig. (4) for $2\mu sec < t < 8\mu sec$). It is still possible to determine a value for $R_0$ such that, together with $C_0$, it produces losses equivalent to the hysteretical ones when driven at same frequency and at the same field as the actuator itself. In Fig. (6) a hysteresis plot (solid line) of a piezoelectric actuator subjected to a sinusoidal cycling voltage is shown. Here, a PZT-5H based unimorph actuator is used with the given values in Table I where $l$, $w$ and $h$ are the length, width and thickness respectively, $E$ is the Young’s Modulus, and $\epsilon$ is the dielectric permittivity. Capacitance is easily determined by the average slope (in a voltage vs. charge plot a pure capacitor would be represented by a straight line), while the resistance is determined after the area surrounded by the hysteresis curve is numerically computed (the resistor’s value is clearly frequency dependent). Both linear resistor and capacitor fitting hysteresis curve will in general depend upon maximum applied field, but this is not an issue since the charge recovery circuit operates at a given voltage (given by square wave specifications).

<table>
<thead>
<tr>
<th></th>
<th>$E$ (GPa)</th>
<th>$\epsilon$ (C/Vm)</th>
<th>$l$ (mm)</th>
<th>$w$ (mm)</th>
<th>$h$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT-5H</td>
<td>61</td>
<td>$3.36 \times 10^{-8}$</td>
<td>16</td>
<td>6</td>
<td>127</td>
</tr>
<tr>
<td>steel</td>
<td>193</td>
<td>×</td>
<td>16</td>
<td>6</td>
<td>76</td>
</tr>
</tbody>
</table>

TABLE I
PARAMETERS OF THE PZT-5H BASED UNIMORPH ACTUATOR WITH A STEEL ELASTIC LAYER FOR THE Hysteresis MEASUREMENTS.

So far, the applicability of linear models to the charge recovery circuit has been justified; the next section will examine results of such an application.

B. Application of General Linear Model for Piezoelectric Actuators to Charge Recovery Circuit

In this section, a prototype charge-recovery circuit used with piezoelectric bimorphs made of two layers of PZT-5H material ($16\text{ mm} \times 6\text{ mm} \times 127 \mu\text{m}$ size) is discussed. Assuming as a general model the linear network in Fig. (5)b, behavior of piezoelectric actuators can be predicted when driven by a charge recovery circuit. Referring to Fig. (5)a, supposing the resonant frequency $\omega_0$ (determined by inductor and piezoelectric
capacitance) to be \( \omega_i < \omega_0 < \omega_{i+1} \), then for any mode \( n > i \) (i.e. whose resonant frequency \( \omega_n \gg \omega_0 \)), the effect of \( L_n \) can be neglected since \( \omega_0 L_n \ll 1/(\omega_0 C_n) \), i.e. \( L_n \) can be replaced with a short circuit. Such higher modes can be simply described by an \( R_n C_n \) branch and together with \( R_0 C_0 \) will determine the impedance at frequency \( \omega_0 \). An estimate for the capacitance of piezoelectric actuator at such frequency is:

\[
\|C(j\omega_0)\| = C_0 + \sum_{n=i+1}^{\infty} C_n
\]

which is valid when losses \( R_i \) are negligible. In the general case, \( R_0 C_0 \) in parallel with \( R_n C_n \) (where \( n > i \)) has to be considered, which is still a frequency dependent lossy capacitor.

The remaining significant modes, which are typically 3–5 and correspond to low frequencies \( \omega_1, \omega_2, \ldots, \omega_i \), are responsible for the oscillations arising soon after switching.

In left side of Fig. (7), the time response (normalized voltage across both piezoelectric layers) of a bimorph actuator is shown. At time \( t < 5 \mu\text{sec} \) one side is fully charged (dashed lines in the upper picture) and the other is fully discharged. When \( t = 5 \mu\text{sec} \), the switch is turned on and charge between the two piezoelectric layers of bimorph actuator starts being exchanged, similar to what happened in Fig. (4). At about \( t = 10 \mu\text{sec} \), around 75% of charge has been transferred (i.e. recovered) and exchange between the two capacitors is over. The lower (left) picture in fact shows how current (obtained by numerically integrating voltage across inductor) can practically be considered zero after \( t = 11 \mu\text{sec} \).

At this point each side of the actuator is disconnected from the other. When real capacitors are considered, voltage is held constant after the diode turns off (as in Fig. (4) for \( t > 8 \mu\text{sec} \)) but when piezoelectric actuators are involved, oscillations arise. Very similar behavior can be observed when two similar unimorphs are used instead of a single bimorph.

As an example, a model was derived in order to fit experimental results. Simulations are shown in the right side of Fig. (7). The model simply comprises an \( R_0 C_0 \) (\( C_0 = 19.9 \text{ nF} \) and \( R_0 = 25 \Omega \)) branch and the first four (low frequency) modes \( (R_i L_i C_i, i = 1, 2, 3, 4) \) whose numerical value can be derived from Table II. Only modes 3 and 4 (respectively corresponding to \( \omega_3 = 2\pi \times 20 \text{ kHz} \) and \( \omega_4 = 2\pi \times 100 \text{ kHz} \)) are visible at the displayed time scale; modes 1 and 2 are not fast enough to produce visible effects.

The previous example was only presented to emphasize how transitions occurring during switching time are slow when compared with higher frequency modes. Thus, the voltage across \( C_n \) for \( n > i \)
TABLE II

SPICE parameters used to fit the behavior of the piezoelectric actuator.

<table>
<thead>
<tr>
<th>parameter</th>
<th>units</th>
<th>mode 1</th>
<th>mode 2</th>
<th>mode 3</th>
<th>mode 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>rad/sec</td>
<td>$2\pi \ 475$</td>
<td>$2\pi \ 2530$</td>
<td>$2\pi \ 20000$</td>
<td>$2\pi \ 100000$</td>
</tr>
<tr>
<td>$C$</td>
<td>nF</td>
<td>1.2</td>
<td>0.4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$Q$</td>
<td>-</td>
<td>15</td>
<td>30</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

can perfectly track voltage across $C_0$ (i.e. voltage at actuator’s electrodes). Transitions are fast when compared with remaining low frequency modes ($\omega_1, \omega_2, \ldots, \omega_i$) so that their effect will only take place after the switching is over. This means that the circuit will in fact only recover charge (and therefore energy) stored in capacitors $C_0$ (which is the dominant one, representing dielectric domain) and $C_n$ such that $\omega_n > \omega_0$. For capacitances associated with lower frequency modes, there will be no recovery. A large capacitance $C_i$, with respect to $C_0$, implies a large amount of energy stored in the mode $i$. A value for the inductor $L$ in Fig. (3) shall be chosen in order to resonate with the piezoelectric actuator at a frequency $\omega_0$ which is low enough to include as many modes as possible or at least the ones which store more energy. Fig. (8) shows the energy stored in each mode $i = 1, 2, 3, 4$ (associated with capacitance $C_i$) and the energy stored in the electrical domain (referred to as $i = 0$ since it is associated with $C_0$) when a voltage $V = 250$ V is applied\(^2\). Fig. (8) also shows the value of the inductance required to resonate at $\omega_0 \approx \omega_i$, where $\omega_i$ represents the frequency of each mode as given in Table II.

In order to recover energy from a mode $i$, the inductor $L$ must resonate at $\omega_0 < \omega_i$. From Fig. (8), it is clear how a lower $\omega_0$ implies a larger $L$. It is possible to notice an exponential growth of the required inductance. Increasing $L$ can be a serious problem in applications where compact size and/or small weight are an issue. Size limits allow one to determine an estimate of $\omega_0$ and, by means of an accurate model, it is possible to predict the amount of energy which is not recoverable.

IV. Summary

In this paper a novel method for driving piezoelectric actuators with low frequency square waves for low power applications, such as a micromechanical flying insect, is presented. After analyzing the relationship among the piezoelectric coupling factor, the actuator energy transmission coefficient, and overall efficiency, attention is focussed on the necessity of recovering electric stored energy in

\(^2\)Operating voltage is not relevant per se since the ratio of energy stored in a mode $i$ and the energy stored in the dielectric domain is proportional to the ratio of $C_i$ over $C_0$ but, especially at high fields, depending on the operating voltage, the best fitting linear model may vary because of inherent nonlinearities.
piezoelectric actuators for boosting efficiency. The inherent theoretical inefficiency of existing charge recovering methods, which simply transfer the load’s energy directly into another capacitor, is pointed out. Alternatively, a general (i.e. valid for any capacitive load) method for charge recovery is presented which exploits highly efficient energy transfer between the capacitive load and an inductor. An implementation of such a method, which adds charge recovery functionality to a standard H-bridge topology by including the aforementioned inductor, two extra switches and two self-timed turning-off diodes, is tested by experiments and simulations. Without any attempt at component optimization, a proof-of-principle circuit is realized which leads to more than 92% of charge (i.e. more than 82% of energy) recovered when the load is represented by standard capacitors and more than 75% of charge (56% of energy) when real piezoelectric bimorph actuators are driven. This result is summarized in Table III for a low voltage driving case of the PZT-5H based bimorph actuator where $l$, $w$ and $h$ represents the length, width and thickness, and $C_0$ is the capacitance of the piezoelectric layer of the actuator. Here, 8.9 $\mu$J electrical energy is recovered from the stored 15.9 $\mu$J energy at a moderately high 40 V driving voltage.

<table>
<thead>
<tr>
<th>$l$</th>
<th>16 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>6 mm</td>
</tr>
<tr>
<td>$h$</td>
<td>127 $\mu$m</td>
</tr>
<tr>
<td>$C_0$</td>
<td>19.9 nF</td>
</tr>
<tr>
<td>$V$</td>
<td>40 V</td>
</tr>
<tr>
<td>$W_{\text{stored}}$</td>
<td>15.9 $\mu$J</td>
</tr>
<tr>
<td>$W_{\text{recovered}}$</td>
<td>8.9 $\mu$J</td>
</tr>
</tbody>
</table>

**TABLE III**

Measured stored and recovered electrical energies of the PZT-5H based bimorph actuator.
Acknowledgements

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REFERENCES


Fig. 1. Charge recovering circuits in case of a positively (a) or negatively (b) pre-charged capacitor. The two circuits can merge together (c) by sharing the same inductor and with addition of two extra pre-charging switches S1 and S4.

Fig. 2. (a) Charge can be transferred from a charged capacitor C1 to a discharged one C2. (b) If C1=C2 then the charge will be completely transferred from C1 to C2.
Fig. 3. Complete circuit for applying charge recovery to a unipolar square wave switching stage. $S_{HL}$, $S_{LL}$, $S_{HR}$, $S_{LR}$ represent a standard H-bridge.

Fig. 4. Experimental data (normalized voltage across capacitors vs. time) acquired from proof of principle prototype of unipolar charge recovering circuit. More than 90% of charge is transferred, corresponding to more than 81% of energy recovery.
Fig. 5. Schematic diagram (a) of absolute value of generalized capacitance vs. frequency and equivalent lumped parameter description (b) for a piezoelectric actuator, each $R_iL_iC_i$ circuit corresponds to a mechanical mode, resonating at $\omega_i = \sqrt{L_iC_i}$. 
Fig. 6. Superposition of hysteresis plots (applied voltage vs. injected charge) for a piezoelectric actuator (solid line) and for a linear $RC$ circuit (dashed line).
Fig. 7. Experimental normalized voltage (upper left) and current (lower left) values can be compared with corresponding simulation results (upper and lower right). In this particular case, a bimorph made out of two $16 \times 6 \times 0.127 \text{mm}^3$ PZT-5H layers was driven with $40 \text{ V}$ square waves. An inductor $L = 220 \mu\text{H}$ was used in the charge-recovery circuit.
Fig. 8. Energy stored in each mode when a voltage $V = 250\, V$ is applied. Vertical arrows represent (in a logarithmic scale) the inductance required to resonate at the given mode.