BF2202 Mathematical Methods for Finance

Academic Year : 2017-18                      Semester : 2
Course Coordinator : William C. H. Leon
Other Instructor(s) :
Pre-requisites : Cambridge G.C.E “O” Level Additional Mathematics and
                 AB1202 Statistics & Analysis*
No. of AUs : 4
Contact Hours : 4 hours per week × 13 weeks

* All students should review the Cambridge G.C.E “O” Level Additional Mathematics and
  AB1202 Statistics & Analysis material before attending the BF2202 Mathematical Methods
  for Finance course.

Course Aims

This course is an introduction to mathematical finance. The course serves to equip students with
basic mathematical knowledge and computational skills needed to solve problems related to
finance. Furthermore, this course lays the foundation for more advanced topics, such as
econometrics, stochastic calculus and optimal control theory, which make finance a very exciting
subject.

Course Description

Modern finance draws upon many fields of mathematics, in particular, algebra, econometrics,
numerical analysis, optimization theory, partial differential equations, probability theory, statistics
and stochastic calculus. The diversity of mathematical skills needed to master finance makes it a
very challenging subject for students.

The mathematical topics covered in this course include matrix algebra, calculus, difference
equation, differential equation, optimization and portfolio mathematics (please refer to the
proposed seminar schedule for details). The topics in the first five weeks of the course introduce
students to the simplest financial market model and explain arbitrage pricing in a discrete time
framework. The topics in the next three weeks lay the foundations for continuous time finance.
The topics in the last four weeks introduce students to optimization and portfolio theory. These
selected topics serve as an introduction to some of the fields of mathematics that modern finance
draws upon.
Course Instructor

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Office</th>
<th>Phone</th>
<th>E-mail</th>
</tr>
</thead>
<tbody>
<tr>
<td>William C. H. Leon*</td>
<td>S3-B1A-29</td>
<td>6790 5647</td>
<td><a href="mailto:achleon@ntu.edu.sg">achleon@ntu.edu.sg</a></td>
</tr>
</tbody>
</table>

* Course co-ordinator.

Learning & Teaching Approach

This course is conducted through 2-hour seminars, held twice per week, commencing in Week 1 and ending in Week 13.

<table>
<thead>
<tr>
<th>Approach</th>
<th>How this approach support students to achieve the learning outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seminar</td>
<td>Interactive seminar sessions provide ample opportunities for open discussion. Questions raised during seminars allow students to think critically, and share their ideas and perspective with their peers. This also allows me to get the concepts clearly through the entire class by involving each student present and ensure that the targeted learning outcomes are being achieved. During in-class discussions, students will also receive immediate verbal feedback about the level of proficiency of their responses.</td>
</tr>
<tr>
<td>Assignments &amp; Quizzes</td>
<td>Students will receive feedback about their assignments and quizzes, and how they might go about improving their understanding.</td>
</tr>
</tbody>
</table>

Course Learning Outcomes

By the end of this course, students will learn to use mathematical techniques to solve asset pricing and portfolio management problems. Specifically, students will be able to:

1. Construct and describe models for financial markets.
2. Analyse and identify arbitrage opportunities.
3. Model and price financial securities.
4. Explain and apply the Black-Scholes-Merton option pricing model.
5. Setup and perform optimization.
6. Construct and manage optimal portfolios.
### Assessment Plan

<table>
<thead>
<tr>
<th>Course Learning Objective</th>
<th>Assessment Method</th>
</tr>
</thead>
</table>
| **Knowledge Acquisition**  
Students demonstrate knowledge and application ability to use mathematical techniques to solve asset pricing and portfolio management problems | Problem discussions.                 |
|                                                                                         | Assignment.                            |
| **Problem Solving & Decision Making**  
Students demonstrate the ability to reason and solve quantitative problems from a wide array of authentic contexts and everyday life situations. They understand and can create sophisticated arguments supported by quantitative evidence and they can clearly communicate those arguments in a variety of formats (using words, tables, graphs, mathematical equations, etc., as appropriate). | Quizzes & final written examination.   |
|                                                                                         | (Refer to Appendix 1 for the assessment rubrics.) |

### Course Assessments

<table>
<thead>
<tr>
<th>Components *</th>
<th>Learning Outcomes Tested</th>
<th>Marks</th>
<th>Individual / Group</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coursework</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>C1:</strong> Assignments</td>
<td>1 – 6</td>
<td>10</td>
<td>Individual</td>
</tr>
<tr>
<td><strong>C2:</strong> Participations</td>
<td>1 – 6</td>
<td>10</td>
<td>Individual / Group</td>
</tr>
<tr>
<td><strong>C3:</strong> Quizzes</td>
<td>1 – 6</td>
<td>20</td>
<td>Individual</td>
</tr>
<tr>
<td>Final examination $</td>
<td>$ 1 – 6</td>
<td>60</td>
<td>Individual</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>100</td>
<td></td>
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</tr>
</tbody>
</table>

* Details will be elaborated in the first teaching week.

$ Refer to Appendix 1 for the assessment rubrics.

**C1:** The marks are allocated to the assignments as specified in the proposed weekly schedule.

Please do not email your assignment to the instructor. The assignments must be typed-written or legibly hand-written, and must be submitted to the instructor in class on the due dates (as set by the instructor).

You should communicate clearly in writing up your submissions for the assignments. Your answers must display your understanding well and be written in a logical, complete, coherent, and well-organized fashion.
Copying solutions for the assignments from unauthorized sources (including from peers and seniors) are serious violations of the university honour codes. If caught, the student will be reported to the undergraduate office and be given an “F” grade for the course.

C2: Absence from class will affect your participation marks.

This course requires you to be in class to participate in activities and discussions. There will be no make-up opportunities for in-class activities. If you will be absent from a seminar session, you must inform your instructor via email prior to the start of the class.

C3: The quizzes are closed-book exams. The intent of the quizzes is to examine how well you understand the technical topics covered in class.

If you have medical conditions that require special arrangements for the quizzes, you must inform your instructor latest by week 4 (i.e., two weeks before the first quiz). Medical proof must be provided.

Students who fail to take the quiz will have a zero score unless they have a valid reason.

Valid reasons are:
- on medical leave (a medical certificate must be submitted);
- representing the school or the university in competition (official letter must be produced);
- bereavement (immediate family members only);
- job interview (allowed only for students in their final year).

The following reasons are invalid:
- hall competition;
- part-time job;
- overslept;
- interpersonal relationship issues;
- traffic jam or train service disruption;
- job interview (applies to students not in their final year);
- any other reasons deemed unacceptable by the instructor.

For students who missed any of the quiz for valid reasons, the instructor will rescale their total coursework marks (based on other coursework components and the instructor’s judged difficulty levels of the quizzes).

§ The final written examination is a restricted-open-book exam where students may bring into the exam hall one A4-size cheat sheet with information on mathematical formulas, computation steps, etc. There is no restriction on the types or formats of information on the cheat sheet. You may use both sides of the A4-paper.

The intent of the final examination is to determine how well you understand the mathematical tools of finance, and if you are able to apply them to novel as well as routine situations.
Readings & References

**BBT**  Jeffrey Baldani, James Bradfield and Robert Turner
*Mathematical Economics*
Thomson / South-Western, 2nd Ed., 2005

**C**  Ales Cerny
*Mathemtical Techniques in Finance*
Princeton University Press, 2004
(Call No. HG106.C415)

(ISBN: 9780691141213 / Call No. XX(1104382.1))

**D**  Kamran Dadkhah
*Foundations of Mathematical and Computational Economics*
South-Western, 2006
(Call No. HB135.D121)

–  Springer, 2011
(ISBN: 9783642137488 / Call No. XX(798198.2))

**LC**  Hon Sing Lee and Gerald H. L. Cheang
*Introduction to Calculus and Matrix Algebra with Applications in Finance*
(Call No. QA303.L478)

**SB**  Carl P. Simon and Lawrence Blume
Mathematics for Economists
Norton, 1994
[https://books.google.com.sg/books?id=cxSaQgAACAAJdq](https://books.google.com.sg/books?id=cxSaQgAACAAJdq)
SH Knut Sydsaeter and Peter Hammond
*Essential Mathematics for Economic Analysis*
http://catalogue.pearsoned.co.uk/educator/product/Essential-Mathematics-for-Economic-Analysis/9780273760689.page

TH John L. Teall and Iftekhar Hasan
*Quantitative Methods for Finance and Investments*
Blackwell Publishing, 2002
https://books.google.com.sg/books?id=OLesQgAACAAJ
# Proposed Weekly Schedule

<table>
<thead>
<tr>
<th>Week (Week beginning)</th>
<th>Topics</th>
<th>Learning Outcomes</th>
<th>Readings &amp; Discussion (10%)</th>
<th>Assignments (10%)</th>
</tr>
</thead>
</table>
| 1                     | Time Value of Money | 1. Arithmetic & Geometric Series.  
2. Simple & Compound Interest.  
3. Limits & Exponential Function.  
4. Continuous Compounding of Interest.  
5. Present & Future Values.  
8. Annuity, Perpetuity & Amortization.  
11. Solving Problems with Excel. | BBT  Chapter 15–16  
D*  Chapter 2 & 14  
LC*  Chapter 2–3  
SB  Chapter 5  
SH  Chapter 10  
TH*  Chapter 4 | Assignment 1 |
2. One-Period Finite State Model.  
4. Securities as Vectors.  
5. Vector Space.  
7. Matrix as a Collection of Securities.  
8. Transposition.  
10. Working with Matrices in Excel. | BBT  Chapter 3–4  
C*  Chapter 1  
D*  Chapter 4  
LC*  Chapter 11–12 & 15  
SH  Chapter 15–16  
TH  Chapter 7 | Assignment 2 |
<table>
<thead>
<tr>
<th>Week (Week beginning)</th>
<th>Topics</th>
<th>Learning Outcomes</th>
<th>Readings &amp; Discussion (10%)</th>
<th>Assignments (10%)</th>
</tr>
</thead>
</table>
2. Hedging.  
3. Linear Independence and Redundant Securities.  
4. The Structure of the Marketed Subspace.  
5. Identity Matrix and Arrow-Debreu Securities.  
6. Matrix Determinant & Inverse.  
7. Inverse Matrix and Replicating Portfolios.  
9. Solving Linear Systems of Equations in Excel | | Assignment 3 (1%) |
2. Asset Prices, Returns & Portfolio Units.  
3. Arbitrage.  
4. No-Arbitrage Pricing.  
5. State Prices & the Arbitrage Theorem.  
6. State Prices and Asset Returns.  
8. State Prices & No-Arbitrage Pricing. | C* Chapter 2  
D Chapter 4  
LC* Chapter 10 & 17  
TH* Chapter 10 | Assignment 4 (1%) |
| 5                    | Arbitrage and Pricing of Securities II | 1. One-Period Binomial Model.  
2. Multi-Period Binomial Model.  
3. Computational Constraints.  
4. Implementing the Binomial Model in Excel. | | Assignment 5 (1%) |
<table>
<thead>
<tr>
<th>Week (Week beginning)</th>
<th>Topics</th>
<th>Learning Outcomes</th>
<th>Readings &amp; Discussion (10%)</th>
<th>Assignments (10%)</th>
</tr>
</thead>
</table>
| 6                     | Univariate Calculus | 1. Functions.  
2. Payoff Functions of Derivatives.  
5. Continuity & Differentiability.  
6. Differentiation.  
7. Product Rule, Quotient Rule & Chain Rule.  
8. Limits & L'Hospital Rule.  
10. Integration.  
11. Integration by Substitutions & by Parts. | BBT Chapter 5–6  
D* Chapter 6–8 & 11  
LC* Chapter 4–6 & 11  
SB Chapter 2–4 & 13–14  
SH Chapter 6–7, 9 & 11–12  
TH* Chapter 8–9 | Assignment 6 (1%) |
| 7                     | Multivariate Calculus | 1. Partial Derivatives.  
2. Total Differential.  
3. Implicit Differentiation.  
4. Fundamental Theorem of Calculus.  
5. Working with Symbolic Math using Computer. | Assignment 7 (1%) |
<table>
<thead>
<tr>
<th>Week (Week beginning)</th>
<th>Topics</th>
<th>Learning Outcomes</th>
<th>Readings &amp; Discussion (10%)</th>
<th>Assignments (10%)</th>
</tr>
</thead>
</table>
| 8                     | Black-Scholes-Merton Option Pricing Model | 1. Differential Equation.  
3. Black-Scholes-Merton Option Pricing Model. | BBT Chapter 15–16  
D Chapter 13  
LC* Chapter 9  
SB Chapter 24–25  
TH* Chapter 10 | Assignment 8 (1%) |
| 9                     | Unconstrained Optimization | 1. Maxima & Minima of Univariate Functions.  
2. First & Second Order Conditions for Optimality.  
4. Unconstrained Optima of Multivariate Functions.  
5. Unconstrained Optimization using Excel. | BBT Chapter 7–10  
D* Chapter 9–10  
LC* Chapter 7  
SB Chapter 17–19 & 21  
SH Chapter 8 & 13–14 | Assignment 9 (1%) |
| 10                    | Constrained Optimization | 1. Optimization with Equality Constraints.  
2. The Lagrangian Function.  
5. Optimization with Inequality Constraints.  
6. Inequality Constraints & Karush-Kuhn-Tucker Conditions.  
7. Constrained Optimization using Excel | | Assignment 10 (1%) |
<table>
<thead>
<tr>
<th>Week (Week beginning)</th>
<th>Topics</th>
<th>Learning Outcomes</th>
<th>Readings &amp; Discussion (10%)</th>
<th>Assignments (10%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Portfolio Mathematics I</td>
<td>1. Returns of Securities.</td>
<td>D Chapter 3 &amp; 5</td>
<td>Assignment 11 (1%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Expected Return &amp; Variance-Covariance Matrix.</td>
<td>LC* Chapter 14</td>
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<td></td>
<td>3. Mean-Variance Framework.</td>
<td>TH* Chapter 6 &amp; 8</td>
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<td>4. Portfolio Expected Return &amp; Variance.</td>
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<td>5. Quadratic Forms.</td>
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<td>6. Basic Portfolio Problems</td>
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<td>7. Minimum Variance Portfolio.</td>
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<td></td>
<td>8. Minimizing Portfolio Risk for a Required Expected Return.</td>
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<tr>
<td></td>
<td></td>
<td>Maximizing Portfolio Expected Return Given Tolerated Risk.</td>
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</tr>
<tr>
<td>12</td>
<td>Portfolio Mathematics II</td>
<td>9. Solving Portfolio Problems by Matrix Operations.</td>
<td></td>
<td>Assignment 12 (1%)</td>
</tr>
<tr>
<td></td>
<td>Quiz 2 covering topics in week 6–11 (10%)</td>
<td>10. Solving Portfolio Problems by Row Operations.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Eigenvalues &amp; Eigenvectors</td>
<td>1. Eigenvalues.</td>
<td>LC* Chapter 13</td>
<td>Assignment 13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Eigenvectors.</td>
<td>SB* Chapter 23</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>4. Diagonalization of Matrices.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The main reference(s).
Academic Expectations

I expect you to complete all assigned pre-class readings and activities, attend all seminar classes punctually, submit all assignments by due dates, and take all scheduled quizzes. I also expect you to take responsibility to follow up with course notes, assignments and course related announcements for seminar sessions you have missed.

I expect you to come prepared for class as detailed in the proposed weekly schedule. As seminar time is precious, it must be used efficiently. You cannot be taught everything during the seminars. Much of your learning must occur outside the seminar room. At least you should plan to study two or more hours on your own for each hour of seminar.

I expect you to read the textbooks for familiarity before the material is discussed in the seminar. When you are prepared, the faster-pace seminar discussion will make more sense. After the seminar, carefully reread the textbooks along with your notes to reinforce your understanding of the material. The textbooks not only gives detailed accounts of the material covered in the course, but they also contain problems to test your understanding. These problems should supplement those you see in the seminar. Try to work through the material and to fill in any omitted steps.

You should attempt all the assignments on your own and try additional problems in areas where you feel you are weak in. It is your responsibility to communicate clearly in writing up your submissions for the assignments. Your answers must display your understanding well and be written in a logical, complete, coherent, and well-organized fashion.

I stand ready to help you learn, but the responsibility is yours. If you are experiencing difficulty, make an appointment to see me during my office hours for extra help. If you do not do your part, then there is very little I can do to make up for it.

Academic Integrity Policy

NTU’s Student Academic Integrity Policy requires all members of the NTU community to uphold the values of academic integrity in all academic undertakings. The policy defines the following acts as academic dishonesty:

- plagiarism,
- academic fraud and
- facilitating academic dishonesty.

All students are expected to read and observe the policy guidelines detailed at this website: http://academicintegrity.ntu.edu.sg/for-everyone/policy/. The academic integrity website also highlights the penalties that will be imposed on students who are found to have violated the policy, and the processes that will be followed when we deal with cases of academic dishonesty.
## Problem Solving & Decision Making Rubric

**Learning Objective:** The ability to reason and solve quantitative problems from a wide array of authentic contexts and everyday life situations. Students demonstrate they understand and can create sophisticated arguments supported by quantitative evidence, and they can clearly communicate those arguments in a variety of formats (using words, tables, graphs, mathematical equations, etc., as appropriate).

<table>
<thead>
<tr>
<th>Traits</th>
<th>Performance</th>
<th>Evaluation: Scant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Substantially Developed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interpretation</strong></td>
<td>Ability to explain information presented in mathematical forms (e.g., equations, graphs, diagrams, tables, words).</td>
<td>Scant</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td></td>
<td>Attempts to explain information presented in mathematical forms, but draws incorrect conclusions about what the information means.</td>
<td></td>
<td></td>
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<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td><strong>Representation</strong></td>
<td>Ability to convert relevant information into various mathematical forms (e.g., equations, graphs, diagrams, tables, words).</td>
<td>Scant</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td></td>
<td>Completes conversion of information but resulting mathematical portrayal is inappropriate or inaccurate.</td>
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<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td><strong>Calculation</strong></td>
<td>Ability to perform calculations.</td>
<td>Scant</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td></td>
<td>Calculations are attempted but are both unsuccessful and are not comprehensive.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td><strong>Application/Analysis</strong></td>
<td>Ability to make judgments and draw appropriate conclusions based on the quantitative analysis of data, while recognizing the limits of this analysis.</td>
<td>Scant</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td></td>
<td>Uses the quantitative analysis of data as the basis for tentative, basic judgments, although is uncertain about drawing conclusions from this work.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td><strong>Presentation</strong></td>
<td>Expressing quantitative evidence in support of the argument or purpose of the work (in terms of what evidence is used and how it is formatted, presented, and contextualized).</td>
<td>Scant</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
<tr>
<td></td>
<td>Presents an argument for which quantitative evidence is pertinent, but does not provide adequate explicit numerical support.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Substantially Developed</td>
</tr>
</tbody>
</table>
Class Participation Rubric

Learning Objective: The ability to engage in open-ended and collaborative exchange of ideas among instructor and students for the purpose of furthering students thinking, learning, problem solving and understanding.

<table>
<thead>
<tr>
<th>Traits</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance</td>
<td>Missed more than 2 classes without valid reason</td>
<td>Missed 1 class without valid reason</td>
<td>Full attendance in class</td>
</tr>
<tr>
<td>Punctuality</td>
<td>Was late for class on more than 2 occasions</td>
<td>Was late for class on 1 occasion</td>
<td>Always on time for class</td>
</tr>
<tr>
<td>Engagement</td>
<td>Hardly focuses in class (e.g. using mobile phone, unnecessary chatting)</td>
<td>Occasionally engages in distracting activities (e.g. using mobile phone, unnecessary chatting) in class</td>
<td>Engages fully in class</td>
</tr>
<tr>
<td>Contribution Frequency</td>
<td>Does not speak up or contribute in class</td>
<td>Occasionally speaks up or contributes in class</td>
<td>Speaks up or contributes in all classes</td>
</tr>
<tr>
<td>Contribution Quality</td>
<td>No contributions or contributions lack substance</td>
<td>Contributions demonstrate knowledge of subject matter</td>
<td>Contributions are constructive and insightful</td>
</tr>
</tbody>
</table>