Optimal Power Allocation for Secondary Users in Cognitive Relay Networks

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Abstract—We consider a cognitive relay network (CRN) where the secondary users (SUs) are involved as cooperative relays in a primary user’s (PU) communication. To avoid generating interference to PU transmissions, it is assumed that SUs can transmit only when the PU’s channel is idle. On one hand, SUs use some relay powers to speed up the PU’s transmissions. Consequently, PU’s buffer will be depleted faster, resulting in more channel idle times (i.e., more transmission opportunities for SUs). On the other hand, due to the energy limit, less power can be used for SU’s own transmissions if it uses too much power on relaying. Thus, an optimal power allocation strategy is necessary to address this tradeoff. In this paper, the power allocation problem for both single-SU case and multiple-SU case is investigated. The former is formulated as a utility maximization problem, while the latter is modeled as a non-cooperative game. The existence and uniqueness of the Nash equilibrium (NE) are proved, and the impact of different system parameters on NE is comprehensively analyzed through numerical results.

Index Terms – Power allocation, cognitive relay network, non-cooperative game, Nash equilibrium.

I. INTRODUCTION

The cognitive radio network is emerging as an attractive solution to address the issues of spectrum shortage and under-utilization. In a cognitive radio network, the unlicensed users (also called secondary users) can opportunistically access the channels (i.e., frequency bands) in the spectrum allocated to the licensed users (also called primary users) when they are found idle. In some cognitive radio networks, the licensed spectrum is treated as a tradable resource that can be leased to secondary users (SU). In return, the SUs may offer some services to improve the PU’s transmission. For instance, consider a scenario where a PU not equipped with multiple antennas (which happens in most cases) is experiencing deep fading on its channels. Suppose the primary user is aware of the SUs in its vicinity, it can use the neighboring SUs to relay its message. In turn, the SUs may agree to participate in the cooperation with PU in exchange for some idle channels. With such arrangement, the PU benefits from the spatial diversity offered by the cooperating SUs whereas the SUs gain by getting their transmission opportunities. This type of network model is referred to as a cognitive relay network (CRN).

Recently, several CRN models were proposed in the literature (i.e., [1]-[5]). In [1], a CRN was proposed in which the PU calculates the optimal time slot duration for leasing its channels to the SUs, to maximize its quality-of-service (QoS) performance. The SUs compete with each other for transmission within the leased time slot. Considering that transmission rate is no longer the focus for the PU when it is already satisfied, [2] proposed an improved framework based on [1], in which the SUs also make a payment proportional to the channel access time allotted by the PU. In [3]-[5], along with the problem of leasing spectrum to the SUs, the issue of power allocation within the CRN has been considered. A novel dynamic spectrum leasing scheme in CRN via power control games involving both PU and SUs was proposed in [3]. With the objective to optimize the CRN performance while limiting the interference measured at the primary receiver, transmit power strategies among the cognitive relays were investigated in [4]. In [5], a cooperative Nash bargaining power-control game was formulated.

In the above works, a common point is that, PU decides how much time the SUs can utilize its channel. In this paper, we consider a different scenario in which the transmission time of the SU is not arranged by PU directly, but depends on when the PU’s channel is idle. With the cooperation from the SUs, the PU’s transmissions rate will be increased. Consequently, the PU’s buffer will be depleted faster, resulting in more channel idle times for the SUs to utilize. Nonetheless, due to the energy limit, if SUs use too much power on relaying, less power will be left for their own transmissions. Hence, a well-designed power allocation scheme is needed to address this tradeoff. However, to the best of our knowledge, none of the existing works has addressed this problem. In this paper, the power allocation problem in the context of both single-SU and multi-SU cases is discussed. The former is formulated as a utility maximization problem while the latter is modeled as a non-cooperative game. The existence and uniqueness of the Nash equilibrium (NE) under specific conditions are proved.

The rest of this paper is organized as follows: In Section II, we provide a detailed description of the system model. The power allocation schemes for both single-SU case and multi-SU case are discussed and the corresponding analysis is presented in Section III. Section IV presents the numerical results. Conclusion and future work are given in Section V.

II. SYSTEM MODEL

A. Network Scenario

As shown in Fig. 1, we consider a cognitive relay network consisting of one primary transmitter (PT) - primary receiver (PR) pair and multiple secondary transmitter (ST) - secondary receiver (SR) pairs equipped with CRs. The SUs can access
the channel of the PU only when the PU\(^1\) has no data to transmit (i.e., the corresponding data queue is empty). In this case, a rational SU should expect the PU’s queue to be empty as soon as possible, which may encourage SU to use some power on relaying PU’s data. Let \(s\) and \(d\) denote the PT and PR, respectively, and \(r_i\) represent \(SU_i\) for ease of presentation. Quasi-static channels are considered in our model. The channels between nodes are assumed to be independent proper complex Gaussian random variables [1], which are considered to be constant within a period of time, but generally varying over different time slots. \(h_{s,d}\) denotes the complex channel coefficient between PT and PR. \(h_{s,r_i}\) and \(h_{r_i,d}\) denote the channel coefficients between PT and \(SU_i\), \(SU_i\) and PR, respectively, \(h_{r_i,s}\) denotes the channel coefficient between \(ST_i\) and \(SR_i\).

In our model, each SU operates in a Decode-and-Forward (DF) mode. For DF transmission, if the SINR is sufficiently high for \(r_i\) to decode the source transmission, then \(r_i\) can serve as a relay for PU \(s\). That is, \(r_i \in D(s)\), where \(D(s)\) is the decoding set of \(s\) [6]. Here we only consider the SUs which can successfully decode the message of \(s\). Therefore, \(D(s) = \{1, 2, \ldots, N\}\) denotes the set of these SUs, where \(N = |D(s)|\). The SUs equally share the idle channel using time-division-multiple-access (TDMA), so that each SU can transmit without others’ interference during its transmission period.

### III. OPTIMAL POWER ALLOCATION SCHEME

#### A. Utility Function Design

When the PU has data to transmit, we apply Protocol II described in [7], in which PU \(s\) communicates with the SUs in \(D(s)\) and PR over the first time slot, while in the second time slot, the SUs forward the received packet to PR. The maximum achievable rate for this protocol in the DF mode is given by [7]:

\[
\mu_s = \min\{R_{sr}^{\max}, R_{coop}^{\max}\}
\]

where \(R_{sr}^{\max}\) and \(R_{coop}^{\max}\) are the achievable rates of the PU-SU link and the cooperative link (including both PT-PR link and ST-PR links), respectively. When all the \(N\) SUs in the decoding set relay for PU \(s\), it is shown in [1] that \(R_{sr}^{\max}\) is dominated by the worst channel. Let \(\sigma^2\) denote the average power of the background thermal noise, then we have

\[
P_{sr}^{\max} = \log_2 \left(1 + \frac{\min_{i \in D(s)} P_s |h_{s,r_i}|^2}{\sigma^2}\right)
\]

where \(P_s\) is the transmission power of the PU. According to [7], we have

\[
R_{coop}^{\max} = \log_2 \left(1 + \frac{P_s |h_{s,d}|^2}{\sigma^2} + \sum_{i \in D(s)} \frac{P_{r_i}^{(t)} |h_{r_i,d}|^2}{\sigma^2}\right)
\]

where \(P_{r_i}^{(t)}\) is the power spent on relaying PU’s data by \(SU_i\).

The average packet arrival rate of the PU is \(\lambda_s\). Packet departure rate \(\mu_s\) can be obtained from (1). Let \(\rho_s = \frac{\lambda_s}{\mu_s}\) denote the traffic intensity of PU \(s\), then the probability for PU \(s\)’s queue to be empty can be derived as [8]:

\[
P(0) = 1 - \rho_s = 1 - \frac{\mu_s}{\lambda_s}
\]

The maximum transmission rate of \(SU_i\) when utilizing the channel of PU can be obtained from

\[
R_i = \log_2 \left(1 + \frac{P_{i}^{(t)} |h_{r_i,s}|^2}{\sigma^2}\right)
\]

where \(P_{i}^{(t)}\) is the power spent on relaying SU \(i\) used to transmit its own data through the channel of PU. The utility function of \(SU_i\) is defined as follows:

\[
U_i = w_1 \mu_s + w_2 R_i
\]

where \(w_1\) and \(w_2\) are the corresponding weights of the transmission rate of PU and \(SU_i\), respectively. The relay power of \(SU_i\) can affect \(\mu_s\) (which further impacts the queue empty probability of PU), while the self-transmission power has direct influence on \(R_i\). Our objective is to find an optimal power allocation to maximize \(U_i\) of \(SU_i\).

#### B. Problem Formulation of Single-PU-Single-SU case

First, we consider a case of single PU and single SU (namely \(SU_1\), i.e., \(N = 1\)). The optimization problem for \(SU_1\) can be formulated as follows:

\[
\begin{align*}
\max_{p_1^{(t)}, p_1^{(s)}} & \quad U_1 = w_1 \mu_s + w_2 R_1, \\
\text{s.t.} & \quad \lambda_s < \mu_s, \\
& \quad \frac{\lambda_s}{\mu_s} P_{i}^{(t)} + \left(1 - \frac{\lambda_s}{\mu_s}\right) P_{i}^{(s)} \leq E_{th}, \\
& \quad 0 \leq P_{i}^{(s)} \leq P_{\text{max}}^{s}, \\
& \quad 0 \leq P_{i}^{(t)} \leq P_{\text{max}}^{s}
\end{align*}
\]

where \(R_1\) is obtained from (5). (8) is the basic constraint for a queue to be stable [9]. Constraint (9) specifies that the energy consumption of \(SU_1\) in unit time duration should be upper bounded by the threshold \(E_{th}\), which is due to the battery life limit of a cognitive node. Constraints (10) and (11) limit the power \(SU_1\) to be within an upper threshold \(P_{\text{max}}^{s}\).

Note that the average achievable transmission rate of PU \(s\) (i.e., \(\mu_s\)) depends on the relationship between \(R_{sr}^{\max}\) and \(R_{coop}^{\max}\). Thus, \(U_1\) has different expressions in following cases:

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1Note that throughout this paper, we use PU and PT interchangeably, so is the case with SU and ST.
Where $SU_1$ uses a relay power which results in $R_{\text{coop}}^\text{max} \geq R_{sr}^\text{max}$, $\mu_s$ becomes a constant equal to $R_{sr}^\text{max}$. Consequently, $U_1$ is only dependent on the self-transmission power of $SU_1$.

- When $SU_1$ uses a relay power resulting in $R_{\text{coop}}^\text{max} \leq R_{sr}^\text{max}$, $\mu_s$ is determined by the relay power and $U_1$ varies with both relay power and self-transmission power of $SU_1$.

Based on above analysis, we have the following results:

**Proposition 1:** For the optimal power allocation strategy, (i.e., the solution of Problem (7)-(11)),

$$P_1(t) = \left[ \frac{E_{th} - \lambda_s / \mu_s \cdot P_1(t)}{1 - \lambda_s / \mu_s} \right] R_{\text{max}}^\text{max},$$

where $\left\lceil \cdot \right\rceil = \min \{ \max \{ 1, l \}, h \}$.

**Proof:** It is obvious that $U_1$ is continuous with $P_1(t)$. By taking partial derivative of $U_1$ with respect to (w.r.t.) $P_1(t)$ it can be shown that the first order derivative of $U_1$ w.r.t $P_1(t)$ is always greater than 0, while the second derivative is always less than 0. Therefore, $U_1$ is a concave increasing function of $P_1(t)$. According to constraint (9), we have

$$P_1(t) \leq \left[ \frac{E_{th} - \lambda_s / \mu_s \cdot P_1(t)}{1 - \lambda_s / \mu_s} \right] R_{\text{max}}^\text{max}. \tag{13}$$

Taking constraint (11) into account, we have

$$P_1(t) = \left[ \frac{E_{th} - \lambda_s / \mu_s \cdot P_1(t)}{1 - \lambda_s / \mu_s} \right] R_{\text{max}}^\text{max}. \tag{12}$$

**Proposition 2:** Assuming that $SU_1$ is rational, it will not choose a relay power resulting in $R_{\text{coop}}^\text{max} > R_{sr}^\text{max}$, if the following condition is satisfied: \(^2\)

$$\max f(P_1(t)) \leq P_{\text{max}}^\text{max} \tag{14}$$

where $f(P_1(t)) = (E_{th} - \lambda_s / \mu_s \cdot P_1(t)) / (1 - \lambda_s / \mu_s)$.

**Proof:** We apply the proof by contradiction to this proposition. Assuming that for the optimal power allocation strategy, $SU_1$ uses relay power $\tilde{P}_1(t)$ resulting in $R_{\text{coop}}^\text{max} > R_{sr}^\text{max}$, then we have $U_1 = w_1 \cdot R_{sr}^\text{max} + w_2 \cdot R_1$. According to Proposition 1, $P_1(t) = \left[ \frac{E_{th} - \lambda_s / \mu_s \cdot P_1(t)}{1 - \lambda_s / \mu_s} \right] R_{\text{max}}^\text{max}$, which is equal to $f(P_1(t))$ when condition (14) is satisfied. Since $\mu_s = R_{sr}^\text{max}$ when $R_{\text{coop}}^\text{max} > R_{sr}^\text{max}$, which is a constant, $f(P_1(t))$ becomes a decreasing function of $P_1(t)$. Thus, its maximum should be achieved when $P_1(t)$ takes its minimum feasible value, i.e., the smallest value satisfying $R_{\text{coop}}^\text{max} > R_{sr}^\text{max}$. Based on our assumption, $\tilde{P}_1(t)$ should be the minimum feasible value of $P_1(t)$.

Note that when condition (14) is satisfied, according to Proposition 1, $P_1(t)$ will always be bounded by $f(P_1(t))$ rather than $P_{\text{max}}$. Consider that if $SU_1$ uses relay power $\tilde{P}_1(t)$ which results in $R_{\text{coop}}^\text{max} = R_{sr}^\text{max}$, it is straightforward to have $f(P_1(t)) > f(\tilde{P}_1(t))$ since $P_1(t) < \tilde{P}_1(t)$. Hence, we can conclude that using relay power $\tilde{P}_1(t)$ instead of $P_1(t)$ can achieve higher utility for $SU_1$, which contradicts with our previous assumption that $\tilde{P}_1(t)$ is the relay power for optimal strategy of $SU_1$. Consequently, $SU_1$ will not choose a relay power resulting in $R_{\text{coop}}^\text{max} > R_{sr}^\text{max}$ when condition (14) is satisfied.

Note that Proposition 2 can be extended to multi-SU case, the proof by contradiction can be applied similarly. Based on Proposition 1 and Proposition 2, we have the knowledge that $P_1(t)$ can be obtained from $P_1(t)$ by $P_1(t) = \left[ f(P_{N_1}) \right] R_{\text{max}}^\text{max}$.

Thus, $U_1$ is only dependent on $P_1(t)$, and we can obtain the maximum value of $U_1$ by taking its derivative w.r.t $P_1(t)$ and equating it to 0. The corresponding optimal power allocation strategies including $P_1(t)$ and $P_1(t)$ can be obtained.

C. **Problem Formulation of Single-PU-Multiple-SU Case**

Next, we consider a more general case of multiple SUs. The power allocation problem for $SU_1$ can be formulated as:

$$\max_{P_1(t), P_1(t)} \quad U_i = w_1 \cdot \mu_s + w_2 \cdot R_i, \tag{15}$$

s.t. \( \lambda_s < \mu_s \), \( \frac{\lambda_s}{\mu_s} P_i(t) + \frac{1}{N} \left( 1 - \frac{\lambda_s}{\mu_s} \right) P_i(t) \leq E_{th}, \tag{16} \)

$$0 \leq P_1(t) \leq P_{\text{max}}, \tag{17}$$

$$0 \leq P_i(t) \leq P_{\text{max}}. \tag{18}$$

where the factor $\frac{1}{N}$ is due to the equal share of the idle channel among $N$ cooperating SUs. For simplicity, it is assumed that all SUs have the same $P_{\text{max}}$ value. Different from that in (7), the utility $U_i$ in (15) depends on not only the strategy of $SU_i$ (i.e., the relay power $P_i(t)$ and self-transmission power $P_i(t)$), but also the other SUs’ strategies, denoted by $(P_{i-1}, P_i)$. Strategies of different SUs are also coupled in constraint (17).

Thus, problem (15)-(19) is not a pure optimization problem but a non-cooperative power allocation game (NPG). The players are the SUs in $D(s)$, and each of which attempts to optimally allocate $P_i(t)$ and $P_1(t)$ for maximizing its own utility $U_i$. The strategy space can be denoted as $P = [P_i]_{i \in D(s)}$, where $P_i = (P_i(t), P_{t-1}(t))$. The utility space is $U = [U_i]_{i \in D(s)}$. Nash equilibrium (NE) is considered to be the solution of this NPG, which is defined as follows:

**Definition 1:** A strategy vector $P^* = (P_1^*, \ldots, P_N^*)$, is a Nash equilibrium (NE) of the NPG $G = [D(s), \{P\}, \{U\}]$ if, for every $i \in D(s)$, $U_i(P_1^*, P_{-i}^*) \geq U_i(P_i^*, P_{-i}^*)$ for all $P_i \in P$, where $U_i(P_1^*, P_{-i}^*)$ denotes the utility of $SU_i$ when taking strategies $P_1^*$, given the other players’ strategies $P_{-i}^*$.

The essence of NE is that, at this point, no player can achieve higher benefit by unilaterally changing its own strategy when the other players keep their strategies unaltered. After investigating this NPG, we have the following result:

**Proposition 3:** Under the assumption that all SUs in $D(s)$ are rational, for the optimal power allocation strategy:

$$P_1(t) = \left[ \frac{(E_{th} - \lambda_s / \mu_s \cdot P_1(t)) \cdot N}{(1 - \lambda_s / \mu_s)} \right] R_{\text{max}}^\text{max}. \tag{20}$$

\(^2\)It is assumed that $\lambda_s < R_{sr}^\text{max}$ and $P_1(t) \leq P_{\text{max}}$ is always guaranteed to assure that the feasible set of Problem (7)-(11) is not empty.
The proof follows the same way as derived for Proposition 1. In the following, we analyze the existence and uniqueness of NE in the NPG.

**Theorem 1:** An NE exists in the NPG \( G = \{D(s), \{P\}, \{U_i\}\} \) defined in (15)-(19), if the following condition is satisfied:

\[
\max_i f(P_i^{(1)}) \leq P_{\text{max}}, \quad \forall i \in D(s). \tag{21}
\]

**Proof:** According to [10], a concave game is defined as a game in which player \( i \) selects an action \( x_i \in \mathbb{R}^{m_i} \) so that the strategy vector \( x = (x_1, \ldots, x_n) \in S \), where \( S \) is a closed bounded convex set. Let \( \phi_i(x) \) denote the payoff function of player \( i \), it should be continuous in \( x \) and concave in \( x_i \), for \( x \in S \).

For our problem, it is easy to verify that the strategy set \( \mathcal{P} \) is a closed bounded convex set, and the utility function \( U_i \) of SU \( i \) is continuous in \( P \). Now, we prove the concavity w.r.t. \( P_i \) by checking the corresponding Hessian matrix. To simplify the notations, let \( a = P_i^0/\sigma_i^2 \), \( b = |h_{i,s}|^2 \), \( c = \sum_{j \in D(s), j \neq i} |h_{j,s}|^2 \), and \( d = |h_{i,t}|^2 \), then \( U_i \) can be denoted as follows:

\[ U_i = w_i^1 \cdot \ln(1 + a + c + b \cdot x) + w_i^2 \cdot \ln(1 + d \cdot y) \tag{22} \]

where \( w_i^1 = \frac{w_{i,i}^1}{\ln(2)} \) and \( w_i^2 = \frac{w_{i,t}^2}{\ln(2)} \). The Hessian matrix of \( U_i \) can be obtained as follows:

\[
H_{U_i} = \begin{bmatrix}
-\frac{b^2 w_i^1}{(1 + a + c + b \cdot x)^2} & 0 \\
0 & -\frac{d^2 w_i^2}{(1 + d \cdot y)^2}
\end{bmatrix}. \tag{23}
\]

It can be found that \( H_{U_i} \) is a square matrix, since the first-order principal minor determinant of \( H_{U_i} \), i.e., the first element of \( H_{U_i} \), is obviously less than 0, and the second-order principal minor determinant of \( H_{U_i} \) is:

\[
\det(H_{U_i}) = \frac{b^2 d^2 w_i^1 w_i^2}{(1 + a + c + b \cdot x)^2 (1 + d \cdot y)^2} > 0.
\]

According to the definition of negative definiteness for a square symmetric matrix [10], \( H_{U_i} \) is negative definite. Thus, \( U_i(P_i) \) is concave in \( P_i \). Consequently, the NPG is a concave game. According to Rosen’s Theorem that a concave game always has at least one NE [11], we can arrive at the conclusion that an NE exists in the NPG \( G = \{D(s), \{P\}, \{U_i\}\} \) under condition (21).

Next, we investigate the uniqueness of NE. The key aspect of the uniqueness proof is to verify that the game model is diagonally strictly concave on its strategy space \( \mathcal{P} \), i.e., for any two strategy vectors \( P(0) \neq P(1) \) with \( P(k) = [P_i(k), \ldots, P_N(k)]^T \) for \( k = 0, 1 \), where \( P_i(k) = (P_i^{(1)}(k), P_i^{(1)}(k)) \), and for \( r = [r_1, \ldots, r_N]^T \) with \( r_i > 0 \) \( i = 1, \ldots, N \), the following inequality holds [11]:

\[
\Delta P(1,0)g(P(0),r) + \Delta P(0,1)g(P(1),r) > 0 \tag{24}
\]

where \( \Delta P(1,0) \) is a simplified denotation of \( (P(1) - P(0))^T \), and \( \Delta P(0,1) \) represents \( (P(0) - P(1))^T \). The function \( g(P,r) \) is defined as follows:

\[
g(P,r) = \left[ r_1 \frac{\partial U_1}{\partial P_1(0)}, \ldots, r_N \frac{\partial U_N}{\partial P_N(0)} \right]^T. \tag{25}
\]

The following theorem addresses the uniqueness of the NE of the NPG.

**Theorem 2:** The NE of the NPG defined in (15)-(19) is unique, when condition (21) is satisfied.

**Proof:** For Condition (24), we have

\[
(\Delta P(1,0)g(P(0),r) + \Delta P(0,1)g(P(1),r))^T \frac{g(P(0),r) - g(P(1),r)}{l} \tag{26}
\]

where

\[
\frac{g(P(0),r) - g(P(1),r)}{l} = \left( P(1) - P(0) \right)^T \left[ r_1 \left( \frac{\partial U_1}{\partial P_1(0)} - \frac{\partial U_1}{\partial P_1(1)} \right), \ldots, r_N \left( \frac{\partial U_N}{\partial P_N(0)} - \frac{\partial U_N}{\partial P_N(1)} \right) \right]^T.
\]

According to the definition of \( \Delta P(1,0) \), \( \Delta P(0,1) \), and the parameters are set as follows: \( \Delta P(1,0) = (P(1) - P(0))^T \), and \( \Delta P(0,1) = (P(0) - P(1))^T \). Let \( \delta_i = (P_i(0) - P_i(1))(\frac{\partial U_i}{\partial P_i(0)} - \frac{\partial U_i}{\partial P_i(1)}) \), and \( \phi_i = (P_i(0) \cdot P_i(1))(\frac{\partial U_i}{\partial P_i(0)} - \frac{\partial U_i}{\partial P_i(1)}) \). According to the first and second order derivatives of \( U_i \) w.r.t. \( P_i(0) \) and \( P_i(1) \), it can be found that when condition (21) is satisfied, \( \frac{\partial U_i}{\partial P_i(0)} \) and \( \frac{\partial U_i}{\partial P_i(1)} \) are monotonically decreasing w.r.t. \( P_i(0) \) and \( P_i(1) \), respectively. Thus, if \( P_i(1) > P_i(0) \), we have \( \frac{\partial U_i}{\partial P_i(0)} - \frac{\partial U_i}{\partial P_i(1)} > 0 \), and, hence, \( \delta_i > 0 \). Similarly, we also have \( \delta_i > 0 \) for \( P_i(1) < P_i(0) \). Using the same method, we can verify that \( \phi_i > 0 \). Consequently, \( \sum_{i=1}^{N} r_i (\delta_i + \phi_i) > 0 \), and thus we arrive at the conclusion that our NPG has a unique NE under condition (21).

To obtain the NE, we apply the best response strategy. Specifically, given the other SUs’ strategies \( P_{-i} \), the optimal strategy of SU \( i \) can be obtained from

\[
P_i^* = \beta(P_{-i}) = \arg \max_{P_i} U_i(P_i, P_{-i}). \tag{27}
\]

IV. NUMERICAL RESULTS

A. Single-PU-Single-SU Case

In this section, we consider the single-PU-single-SU case and the parameters are set as follows: \( \lambda_s = 1, h_{s,d} = 0.5, h_{s,r_1} = 1, h_{r_1,d} = 0.6, h_{r_1,1} = 0.7, \sigma^2 = 1, \) and \( P_s = 40 \). The maximum power for an SU is \( P_{\text{max}} = 100 \) and \( E_{\text{th}} = 20 \). With these parameters, \( R_{\text{max}} = 5.3576 \). First, we investigate the impact of different weight settings of utility function on the power allocation. Note that different weight settings can reflect the emphasis on either PU or SU. We keep \( w_2 \) fixed to be 1 and increase \( w_1 \) from 0 to 4. The variation of relay power and self-transmission power of SU \( i \) for the optimal power allocation strategy is shown in Fig. 2.
Fig. 3. (a) Transmission rate of PU vs. $P_2^{(t)}$, (b) transmission rate of $SU_2$ vs. $P_2^{(t)}$, and (c) utility of $SU_2$ vs. $P_2^{(t)}$.

In Fig. 2, as $w_1$ varies from 0 to 4, the optimal relay power $P_1^{(t)}$ increases from 0 to 52.61, and the PU transmission rate increases from 3.4594 to 4.9040. The self-transmission power $P_2^{(t)}$ decreases from 35.31 to 5.63, which results in the decrease of SU self-transmission rate from 4.1939 to 1.9102. The reason is that, when $w_1$ is small compared with $w_2$, the PU transmission rate does not contribute much to the $SU_1$’s utility, thus $P_1^{(t)} = 0$ for the optimal strategy at first. This indicates that the SU will not waste power on relaying but to concentrate on improving its self-transmission rate to maximize the utility. However, as $w_1$ keeps increasing with $w_2$ unchanged, the PU transmission rate gradually becomes a major concern for the SU to achieve higher utility. Hence, it is beneficial to use higher relay power for increasing PU transmission rate, although its self-transmission rate decreases at the same time, it will not deteriorate the utility significantly due to the effect of weight.

B. Single-PU-Two-SU Case

1) The Best Response Strategy: We consider the single-PU-multiple-SU case where $N = 2$. The two-SU case is taken as an example since the NE can be graphically shown by the intersection of best response curves. The parameters are set as follows: $\lambda_s = 1.5$, $h_{s,d} = 0.5$, $h_{s, r_1} = h_{s, r_2} = 1$, $h_{r_1,d} = h_{r_2,d} = 0.6$, and $h_{r_1,1} = h_{r_2,2} = 0.7$. The weights $w_1$ and $w_2$ are set as 2 and 1, respectively.

According to the definition of Nash equilibrium, each SU has made best response to the given power allocation of the other SU. To specify how the best response strategy behaves, suppose the power allocation of $SU_1$ have been given such that $P_1^{(t)} = 10$, $P_1^{(t)} = 20$, $SU_2$ will set its relay power $P_2^{(t)}$ and self-transmission power $P_2^{(t)}$ to 31.51 and 29.2, respectively. This is illustrated in Fig. 3 which shows the variation of $SU_2$’s utility with the relay power that $SU_2$ may select.

As shown in Fig. 3(a), when $SU_1$ keeps its power allocation fixed, as the relay power of $SU_2$ increases, the PU transmission rate $\mu_s$ also increases, which indicates that the PU can achieve a higher transmission rate and its buffer will be depleted faster. In Fig. 3(b), it can be found that the self-transmission rate of $SU_2$ gradually decreases. The reason is that as the relay power increases, the self-transmission power decreases simultaneously due to the energy constraint. Since the utility of $SU_2$ depends on both PU transmission rate and self-transmission rate of $SU_2$, it increases first and then decreases, achieving the maximum of 13.33 when $P_2^{(t)} = 31.48$ and $P_2^{(t)} = 29.17$, as shown in Fig. 3(c). The best response strategy aims at searching for this point at which the utility is maximized. In other words, each SU will always make the optimal decision to maximize its own benefit, given the decisions of the other SUs.

2) The Effect of $h_{s,d}$ on the NE: To study the effect of $h_{s,d}$ on NE, we set $\lambda_s = 1.5$, $P_s = 10$, $E_{th} = 15$, $h_{r_1,d} = h_{r_2,d} = 0.4$, and vary $h_{s,d}$. The weights are set as $w_1 = w_2 = 1$, and remain the same for the rest of simulations. As shown in Fig. 4(a), at the NE (corresponding to the intersection of the two best response curves for the same parameter setting), the relay powers of $SU_1$ and $SU_2$ are 7.49 when $h_{s,d} = 0.1$, while those of $SU_1$ and $SU_2$ when $h_{s,d} = 0.3$ and $h_{s,d} = 0.5$ are 5.62 and 1.96, respectively. It can be observed that when the direct channel between the PT and PR is in good condition, both SUs can use less power on relaying, while more power can be used for their own transmissions.

3) The Effect of $h_{r,d}$ on the NE: To study the effect of $h_{r,d}$ on NE, we set $\lambda_s = 1.5$, $h_{s,d} = 0.3$, $h_{r_1,d} = 0.4$ and vary $h_{r_2,d}$. As shown in Fig. 4(b), at the NE, the relay powers of
SU1 and SU2 are 6.8 and 6.64, respectively, when $h_{r_2,d} = 0.3$. The relay powers of SU1 and SU2 when $h_{r_2,d} = 0.4$ are both 5.62. When $h_{r_2,d} = 0.5$, at the NE, the relay powers of SU1 and SU2 are 4.52 and 4.71, respectively. From the results, we observe that when the ST-PR channel gain is improved, SUs can use less power on relaying. Moreover, the SU with a better channel condition to PR (compared with others) will use more relay power. The reason is that, with the same amount of power, the SU with a better relay channel can contribute more to the PU transmission rate, which has a more impact on the improvement of both SUs’ utilities.

4) The Effect of $\lambda_s$ on the NE: To study the effect of $\lambda_s$ on NE, we set $h_{s,d} = 0.3$, $h_{r_1,d} = h_{r_2,d} = 0.4$ and vary $\lambda_s$. As shown in Fig. 4(c), at the NE, the relay powers of SU1 and SU2 are both 2.02, when $\lambda_s = 1$. The relay powers of SU1 and SU2 when $\lambda_s = 1.5$ are 5.62. When $\lambda_s = 2$, the relay powers of SU1 and SU2 are 9.49. From the results, we can conclude that as $\lambda_s$ increases, the SUs should use more power on relaying for the PU to satisfy the constraint $\mu_s > \lambda_s$. The results also accord with our expectation, since when the packet arrival rate at the PU increases, it is rational for the SUs to use more relay power on cooperation for emptying the PU’s queue faster. Consequently, more self-transmission opportunities can be obtained. Otherwise, the PU always has data to transmit, thus the SUs can have seldom chances to access the PU’s channel.

V. CONCLUSION

In this paper, we have considered cooperation in cognitive relay networks from a new perspective. In our scheme, SUs are allowed to choose their relay power and self-transmission power separately. For single-SU case, the power allocation has been formulated as an optimization problem, and the method to solve the optimization problem has been provided. In the case of multiple SUs, a non-cooperative game model has been formulated and the Nash equilibrium (NE) has been achieved. The existence and the uniqueness of the NE under certain conditions have been proved. The effect of different weight settings on the utility function has been discussed. Moreover, the variation in NE with respect to different system parameters has also been presented.

In the future, we plan to generalize the problem to multiple PUs and multi-channel case. The effect of data queues at the SUs will also be investigated. Additionally, we would further look into the modeling of cooperative game which allows some SUs to form a coalition for higher benefit.

REFERENCES