Coalition Formation Games for Relay Transmission: Stability Analysis under Uncertainty

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Abstract—Relay transmission or cooperative communication is an advanced technique that can improve the performance of data transmission among wireless nodes. However, while the performance (e.g., throughput) of a source node can be improved through cooperation with a number of relays, this improvement comes at the expense of a degraded performance for the relay nodes due to the resources that they dedicate for helping the source node in its transmission. In this paper, we formulate a coalitional game among the wireless nodes that seek to improve their performance by relaying each other’s data. The game is classified as a coalition formation game in which the nodes can take individual and distributed decisions to join or split from a given coalition while ensuring that their individual throughput is maximized. A Markov chain model is proposed to investigate the stability of the resulting coalitional structures. Further, we consider the practical case in which the wireless nodes do not have an exact and perfect knowledge of the parameters (e.g., channel quality) in coalition formation. For this scenario, we analyze the stability of the partitions resulting from the proposed coalition formation game under uncertainty. We also define the conditions needed for obtaining the stable and unstable coalitional structures among the nodes that are performing cooperative transmission.

Keywords – Cooperative communications and networking, coalitional game theory, Markov chain.

I. INTRODUCTION

Cooperative communication has recently emerged as a novel communication paradigm that can significantly improve the performance of wireless communication systems [1]. It is proposed as a potential solution to deal with the difficulty in the implementation of Multiple-Input-Multiple-Output (MIMO) systems, where multiple antennas are required at both transmitter and receiver. In particular, cooperative communication attempts to benefit from the broadcast nature of wireless channels to improve the performance of wireless data transmission. In this context, the signals transmitted from a source to a destination can also be overheard by other nodes in the vicinity of the source, which may act as relays (virtual antennas) to process and re-transmit the received signals. In a cooperative network, the nodes can act as both source and relay. That is, whenever a node wishes to transmit data, the other nodes who decide to cooperate will perform cooperative transmission for relaying the data of the node. In turn, when another node in the network wants to transmit its data, the node which previously acted as a source, can now act as a relay and perform cooperative transmission. A key design challenge in cooperative communication is to decide which node should be cooperative and in which cooperative group, especially when the network’s nodes act in a rational and selfish manner, i.e., in a way to maximize their own performance (e.g., throughput). Thus, it is of interest to design cooperative strategies to model how the nodes can act strategically for cooperating (or not cooperating) to relay each other’s data.

In this paper, the problem of coalition (i.e., group) formation for rational nodes with relay transmission in a cooperative network is formulated using the framework of coalitional game theory. In the considered game, each node can autonomously decide to join or split from a coalition while aiming to maximize its own throughput. The decision of each node is contingent on whether, by joining a particular coalition, a gain in performance can be witnessed due to relay transmission by the other nodes in the same coalition. The gain in performance due to cooperative transmission comes at a cost that is a function of the resources (e.g., time slots) that a given node needs to allocate for relaying the data of the other nodes in the same coalition. We analyze the stability of the network partitions resulting from the proposed coalition formation game using a Markov chain model. The stationary probability of the defined Markov chain is used to determine the stable coalitional structures, i.e., the stable coalitions that the nodes can form. Further, we consider the uncertainty in the coalition formation process due to the fact that the nodes may be unaware of some parameters needed for cooperation, e.g., the channel quality. The conditions for the stability of the coalition formation process are defined based on the upper and lower bounds of the stationary probability of the derived Markov chain.

The rest of this paper is organized as follows: Related works are reviewed in Section II. Section III describes the system model and assumptions. Section IV presents the coalitional game formulation for relay transmission. The stability analysis under uncertainty is also presented. Section V presents the numerical results. The summary is given in Section VI.

II. RELATED WORKS

Game theory has been applied to solve various issues in cooperative communications [2]. In [3], a non-cooperative game model was presented to investigate the cooperation among
nodes using decode-and-forward (DF) cooperative transmission. The Nash equilibrium is identified under Rayleigh fading channels as a two-state Markov model. In [4], a Nash bargaining game was formulated to study the bandwidth allocation problem between a source node and a number of relay nodes. Also, the authors studied the conditions under which the source and relay nodes will cooperate. The relay selection and power control problems were formulated in [5] as a two-level Stackelberg game. In this game, the source node, considered as a buyer, pays to the relay nodes to provide them with an incentive to cooperate. In [6], coalitional game theory was used, in combination with cooperative transmission, to solve an important problem related to the boundary nodes in an ad hoc packet forwarding network. A grouping algorithm for relay selection is proposed in [7] to minimize transmit power. Also, an optimal rate allocation scheme among the relay nodes was studied.

Although existing literature tackled several aspects of cooperative transmission using game theory, none of these works considered the problem of performing coalition formation among a number of rational nodes that seek to cooperate for performing cooperative transmission and relaying. One somehow related work is done in [8] where a coalitional game framework was proposed for cooperative communications. However, no analysis on the stability of the resulting coalitional structure was considered. Also, the uncertainty accompanying the coalition formation process for performing relay transmission was ignored.

III. SYSTEM MODEL AND ASSUMPTIONS

![Fig. 1. System model for relay transmission with coalitions {1, 2, 3} and {4}.](image)

A. Network Model

We consider a group of nodes denoted by $\mathcal{N} = \{1, \ldots, N\}$ where $N$ is the total number of nodes. Each node needs to transmit data to a given destination (e.g., a base station as shown in Fig. 1). To improve their performance, the nodes can decide to cooperate and form a group, i.e., a coalition $S \subseteq \mathcal{N}$, in which the nodes relay each other’s data and perform cooperative transmission.

In this paper, we consider that the nodes in a given coalition perform relaying and benefit from cooperative diversity based on a decode-and-forward strategy as in [9]. However, the approach that we propose in the rest of this paper can easily accommodate other strategies (e.g., amplify-and-forward). In the first phase of the cooperative diversity scheme, the source node $i$ transmits using a particular adaptive modulation and coding (AMC) mode while the relay nodes $j \in S$ for $j \neq i$ and the destination receive the signals. In the second phase, the relay nodes repeat the transmission with the same AMC mode while the source node $i$ remains silent. At the end of the second phase, the destination of node $i$ achieves a gain in SNR by combining the space-time decoded signals with those received in the first phase. Let $\gamma_i$ and $\gamma_j$ denote the instantaneous SNR from source node $i \in S$ to its destination and from relay node $j$ to the destination of node $i$ for all $j \in S$ and $j \neq i$, respectively. For the case of multiple relays with the perfect channel between source and relay, the post-processing SNR at the destination of node $i$ can be expressed as follows:

$$\gamma_i^{\text{post}} = \gamma_i + \sum_{j \in S \setminus \{i\}} \gamma_j.$$  

When considering Rayleigh fading channels, the cumulative distribution function (CDF) of the post-processing SNR for single relay node $j$ (i.e., $S = \{i,j\}$) is given by

$$F(\gamma) = \frac{\tau_i}{\gamma_i - \tau_j} (1 - e^{-\tau_j}) + \frac{\tau_j}{\gamma_j - \tau_i} (1 - e^{-\tau_i}),$$  

where $\tau_i$ and $\tau_j$ are the corresponding average SNRs from source node $i$ to its destination and from relay node $j$ to the destination of node $i$. For the multiple-relay case, (2) can be extended as follows [9]:

$$F(\gamma) = \left( \prod_{j \in S \setminus \{i\}} \frac{\tau_i}{\gamma_i - \gamma_j} \right) I + \sum_{j \in S \setminus \{i\}} \left( \frac{\tau_j}{\gamma_j - \gamma_i} \prod_{j' \in S \setminus \{i,j\}} \frac{\tau_j}{\gamma_j - \gamma_j'} \right) B_j,$$

where $|S| \geq 3$, $I = 1 - e^{-\tau_i}$, and $B_j = 1 - e^{-\tau_j}$. Note that for DF based relay transmission, the choice of an AMC mode only depends on the post-processing SNR at the corresponding destination. Given the available AMC modes as well as the minimum required SNR threshold $\Gamma_r$ for each mode, the probability of using mode $r$ for source node $i$ in a given coalition $S$ can be calculated as $\alpha_{i,r}(S) = F(\Gamma_{r+1}) - F(\Gamma_r)$. The transmission rate of a source node $i \in S$ can be obtained from

$$R_i(S) = \sum_{r \in R} \rho_r \alpha_{i,r}(S),$$

where $R$ is a set of AMC modes, and $\rho_r$ is the transmission rate of AMC mode $r$ in packets/time slot.

B. Relay Transmission

Time division multiple access (TDMA) is considered in which a frame is divided into multiple time slots and each node

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1The destination of the nodes is not necessarily the same.
transmits in an allocated time slot. When node $i \in S$ transmits data, the other nodes in the same coalition (i.e., $j \in S$ where $j \neq i$) receive data from node $i$. Then, nodes $j$ relay the data to the destination of node $i$ by transmitting in their allocated time slots.

Due to this relay transmission, each node $i \in S$ will be able to transmit for every $|S|$ frame, where $|\cdot|$ denotes the cardinality. Therefore, the throughput of node $i \in S$ can be calculated from

$$\phi_i(S) = \frac{R_i(S)}{|S|}. \quad (5)$$

By clearly inspecting the defined model, we can see that there exists an interesting tradeoff that governs the decision of the nodes on whether or not it is beneficial to establish a coalition. As more nodes join a coalition, the rate per transmission (i.e., $R_i(\cdot)$) becomes higher due to the better quality of the received signal (more diversity). However, the nodes would have less time to transmit since the allocated time slots have to be used for relay transmission of the other nodes in the same coalition. As a result, the throughput $\phi_i(\cdot)$ may decrease. To efficiently analyze this tradeoff and devise adequate cooperative strategies, a coalitional game formulation will be presented in the next section.

IV. RELAY TRANSMISSION AS A COALITIONAL GAME

In this section, we present a cooperative model based on coalitional game theory to analyze the coalition formation process among the relay nodes and its stability given the selfish nature of each node, i.e., its objective to maximize its own throughput.

A. Players, Payoffs, and Actions

The problem of cooperative relaying mentioned in Section III can be formulated as a coalition formation game with the players being the relay network’s nodes. The set of all players, i.e., nodes, is denoted as $\mathcal{N}$. The payoff received by each node due to relay transmission is captured using the throughput as expressed in (5). The action of each player is to form coalition. A coalition is denoted by $S_x$, where $x$ is an index of coalition. Given a set of current coalitions (i.e., state), the players can decide to join or split from any coalition in a way that can maximize their individual payoffs as follows:

- **Joining**: Let $\mathcal{M}_{in}$ denote a set of candidate coalitions that can join together to form a new single coalition $S_{x'}$. If all nodes $j \in S_x \in \mathcal{M}_{in}$ can gain higher individual payoffs, i.e.,

$$\phi_j(S_{x'}) > \phi_j(S_x) \quad \forall j \in S_x, \quad (6)$$

where $S_{x'} = \bigcup_{S_x \in \mathcal{M}_{in}} S_x$, then the coalitions can decide to join together.

- **Splitting**: Given a coalition $S_x$, the players in this coalition can split (i.e., be partitioned) into multiple new coalitions $S_{x'}$, if all the players $j \in S_x$ can gain higher individual payoffs, i.e.,

$$\phi_j(S_{x'}) > \phi_j(S_x) \quad \forall j \in S_x, \quad (7)$$

where $S_x = \bigcup_{S_x' \in \mathcal{M}_{sp}} S_{x'}$ and $\mathcal{M}_{sp}$ is a set of new coalitions.

We note that this cooperative game formulation has a non-transferable utility (NTU) since the value (i.e., payoffs which is here proportional to the throughput of the node) of a coalition cannot be transferred arbitrarily among players in the same coalition as each individual node has its own payoff. The distributed algorithm using which the nodes can join or split can be implemented as in [10].

B. A Markov Model for Coalition Formation

To analyze the stability of the considered coalition formation game, a Markov chain model can be used [11]. The state of the Markov chain is a partition or a coalitional structure which can be defined as follows: A partition or a coalitional structure is a group of coalitions that span all the players in $\mathcal{N}$ and is defined as $\omega = \{S_1, \ldots, S_{x}, \ldots, S_X\}$ where $S_x \cap S_x' = \emptyset$, for $x \neq x'$, and $\bigcup_{x=1}^{X} S_x = \mathcal{N}$, and $X$ is the total number of coalitions in structure $\omega$, i.e., $X = |\omega|$. Therefore, the state space of Markov chain is defined as follows:

$$\Omega = \{\omega_y|y = \{1, \ldots, D_N\}\}, \quad (8)$$

where $\omega_y$ represents a coalitional structure (spanning all players), $D_N$ is the Bell number obtained from

$$D_i = \sum_{j=0}^{i-1} \binom{i-1}{j} D_j, \quad \text{for } i \geq 1, \text{ and } D_0 = 1. \quad (9)$$

The transition probability matrix of this Markov chain is denoted by $P$ whose elements are $p_{\omega,\omega'}$. Each element $p_{\omega,\omega'}$ represents the probability that the coalitional structure (i.e., state) of all players changes from $\omega$ to $\omega'$. Let $\mathcal{C}_{\omega,\omega'}$ denote the set of candidate players who are bound to make a coalition formation decision which will result in the change of the coalitional structure from $\omega$ to $\omega'$. This transition probability can be obtained from

$$p_{\omega,\omega'} = \begin{cases} \prod_{i \in \mathcal{C}_{\omega,\omega'}} \delta \beta_i(\omega'|\omega), & \omega \rightarrow \omega', \\ 0, & \text{otherwise}, \end{cases} \quad (10)$$

where $\omega \rightarrow \omega'$ is a feasibility condition. In particular, if a coalitional structure $\omega'$ is reachable from $\omega$ given the decision of all players, then the condition $\omega \rightarrow \omega'$ is true. Otherwise, condition $\omega \rightarrow \omega'$ becomes false. $\delta$ is the probability that the players make a decision (e.g., $\delta = 0.5$). $\beta_i(\omega'|\omega)$ is the best-reply rule of player $i$. That is, $\beta_i(\omega'|\omega)$ is the probability that the player $i$ changes decision, and, hence, the coalitional structure changes from $\omega$ to $\omega'$. This best-reply rule is defined as follows:

$$\beta_i(\omega'|\omega) = \begin{cases} \hat{\beta}, & \text{if } \phi_i(S_x^i | \subset \omega') > \phi_i(S_x^i | \subset \omega), \\ \epsilon, & \text{otherwise}, \end{cases} \quad (11)$$
where \( 0 < \hat{\beta} \leq 1 \) is a constant, \( \epsilon \) is a small number that corresponds to the probability that a player makes an irrational decision. Further, we consider that a player can make an irrational coalition formation decision due to either: (i) a lack of information or, (ii) a need for “exploration” in the learning process. In this case, the state transition probability \( p_{\omega\omega'} \) is determined from the product of the transition probabilities of players who do and do not make decisions.

C. Stable Coalition Formation

The solution of the coalition formation game is the coalitional structure, i.e., \( \omega^* \), which can exhibit internal and external stability notions [11]. Internal stability implies that, given a coalition, no player in this coalition has an incentive to leave this coalition and act alone (non-cooperatively as a singleton), since the payoff any player receives in the coalition is higher than that received when acting non-cooperatively. External stability implies that, in a given partition, no player can improve its payoff by switching its current coalition and join another one. In particular, a coalitional structure \( \omega^* \) is said to be stable, if the conditions for internal and external stability are verified for all the coalitions in \( \omega^* \). A stable coalitional structure can be identified from the stationary probability of the Markov chain defined with state space in (8) and transition probability in (10). If the transition probability \( p_{\omega\omega'} \) is exactly known, the stationary probability of the Markov chain can be obtained by solving

\[
\pi^T P = \pi^T, \quad \text{and} \quad \pi^T \mathbf{1} = 1, \tag{12}
\]

where \( \pi = \left[ \pi_{\omega_1}, \ldots, \pi_{\omega_\nu}, \ldots, \pi_{\omega_D} \right]^T \) is a vector of stationary probabilities and \( \pi_{\omega} \) is the probability that the coalitional structure \( \omega \) will be reached.

If the probability of irrational decisions approaches zero (i.e., \( \epsilon \to 0^+ \)), there could be an ergodic set \( E \subseteq \Omega \) of states \( \omega \) in the Markov chain defined by the state space in (8) and the transition probability in (10). This ergodic set \( E \) exists if \( p_{\omega\omega'} = 0 \) for \( \omega \in E \) and \( \omega' \not\in E \), and no nonempty proper subset of \( E \) has this property. In this regard, the singleton ergodic set is the set of absorbing states.

Once all players reach the state (i.e., coalitional structure) in an ergodic set, they will remain in this ergodic set forever. In particular, players will stop making any new decisions for joining or splitting from any coalition. Therefore, the absorbing state is referred to as the stable coalitional structure \( \omega^* \). With this stable coalitional structure, no player has an incentive to change the decision given the prevailing coalitional structure.

D. Stable Coalition Formation under Uncertainty

In practice, there is an uncertainty in the coalition formation process due to the fact that some system parameters have to be estimated for relay transmission (e.g., channel quality and best-reply rule). In particular, the transition probability \( p_{\omega\omega'} \) of the Markov chain can be inaccurate. To analyze the stability of coalition formation under parameter uncertainty, we will adopt an analysis based on a Markov chain with uncertainty [12]. To obtain the stationary probability of a Markov chain with uncertainty, the following optimization problem can be defined:

\[
\min_{\pi, A} \bar{c}^T \pi, \tag{13}
\]

subject to

\[
A \in U, \tag{14}
\]

\[
\mathbf{1}^T A = \mathbf{0}^T, \quad A \bar{\pi} = \bar{0}, \tag{15}
\]

\[
\mathbf{1}^T \pi = 1, \quad \bar{\pi} \geq \bar{0}, \tag{16}
\]

where \( U \) is the uncertainty set of matrix \( A \) and \( \bar{c}^T \) is a vector of objective coefficients. Let \( a_{\omega\omega'} \) denote the element of matrix \( A \) corresponding to coalitional states \( \omega \) and \( \omega' \). The off-diagonal element of matrix \( A \) must be non-positive, i.e., \( a_{\omega\omega'} \leq 0 \), for \( \omega \neq \omega' \) and \( \omega, \omega' \in \Omega \). The diagonal elements must be bounded by one, i.e., \( a_{\omega\omega} \leq 1 \) for \( \omega \in \Omega \).

The interval-value uncertainty set \( U \) is examined. In particular, let \( p_{\omega\omega'}^- \) and \( p_{\omega\omega'}^+ \) denote the lower and upper bounds on the transition probability of the Markov chain used for coalition formation, respectively. The corresponding lower and upper bounds of the elements of the matrix \( A \) can be obtained from \( a_{\omega\omega'}^- = -p_{\omega\omega'}^- \) and \( a_{\omega\omega'}^+ = -p_{\omega\omega'}^+ \), where \( a_{\omega\omega'}^- \leq a_{\omega\omega'} \leq a_{\omega\omega'}^+ \) for \( \omega \neq \omega' \) and \( \omega, \omega' \in \Omega \). Therefore, the conditions for the upper and lower bounds of the elements of the matrix \( A \) are as follows:

- For off-diagonal element: \(-1 \leq a_{\omega\omega'}^- \leq a_{\omega\omega'}^+ \leq 0 \) for \( \omega \neq \omega' \) and \( \omega, \omega' \in \Omega \).
- For diagonal element: \( 0 < a_{\omega\omega}^- \leq a_{\omega\omega}^+ \leq 1 \) for \( \omega \in \Omega \).
- For all elements with \( \omega' \in \Omega \): \( \sum_{\omega \in \Omega} a_{\omega\omega'} \leq 0 \) and \( \sum_{\omega \in \Omega} a_{\omega\omega'}^+ \geq 0 \).

With the interval uncertainty set \( U \), the constraint in (14) becomes

\[
A^- \leq A \leq A^+, \tag{17}
\]

where \( A^- \) and \( A^+ \) are matrices of lower bound \( a_{\omega\omega'}^- \) and upper bounds \( a_{\omega\omega'}^+ \), respectively. Then, the equivalent linear programming problem with the objective defined in (13) and the constraints defined in (15), (16), and (17) can be written as follows:

\[
\min_{\pi, \Xi} \bar{c}^T \pi, \tag{18}
\]

subject to

\[
\pi_{\omega'} a_{\omega\omega'}^- \leq \xi_{\omega', \omega} \leq \pi_{\omega'} a_{\omega\omega'}^+, \quad \omega, \omega' \in \Omega, \tag{19}
\]

\[
\mathbf{1}^T \Xi = \mathbf{0}^T, \quad \Xi \mathbf{1} = \bar{0}, \tag{20}
\]

\[
\mathbf{1}^T \pi = 1, \quad \bar{\pi} \geq \bar{0}, \tag{21}
\]

where \( \xi_{\omega', \omega} \) is the element of matrix \( \Xi \). Let \( \pi^- \) and \( \pi^+ \) denote the smallest and largest possible values (i.e., lower and upper bounds) for the stationary probability of the coalitional structure \( \omega \). \( \pi^- \) can be obtained by solving the optimization problem defined in (18) with the objective coefficient vector \( \bar{c} = \bar{c}' \), where \( \bar{c}' \) is the unit vector (i.e., all elements are zeros except row corresponding to coalitional state \( \omega \) to be one). Further, for \( \pi^+ \), the objective coefficient vector is \( \bar{c} = \mathbf{1} - \bar{c}' \).
The lower and upper bounds of the stationary probability can be used to determine the stability conditions of the coalitional structure $\omega$ given that the irrational decisions approach zero (i.e., $\epsilon \to 0$). Also, the transition probability of the Markov chain used for coalition formation takes a value from $[p_\omega^-, p_\omega^+]$ for $\omega, \omega' \in \Omega$.

**Definition 1 (Stable Coalitional Structure).** If $\pi_\omega^- > 0$, the coalitional structure $\omega$ will be always stable. If $\pi_\omega^+ > 0$, then there exists a case in which the coalitional structure $\omega$ is stable.

**Definition 2 (Unstable Coalitional Structure).** If $\pi_\omega^- = 0$, the coalitional structure $\omega$ will never be stable. If $\pi_\omega^+ = 0$, there exists a case in which the coalitional structure $\omega$ will not be stable.

Since $\pi_\omega^-$ corresponds the minimum stationary probability given that the transition probability $p_\omega^-$ is in $[p_\omega^-, p_\omega^+]$, if $\pi_\omega^- > 0$, the coalitional structure $\omega$ will be always stable. However, if $\pi_\omega^- = 0$, it just indicates that there is at least one case in which the coalitional structure $\omega$ is unstable. Similarly, since $\pi_\omega^+$ is the maximum stationary probability, if $\pi_\omega^+ > 0$, it only indicates that there is at least one case in which the coalitional structure $\omega$ is stable. However, if $\pi_\omega^+ = 0$, the coalitional structure $\omega$ will never be stable.

### V. Performance Evaluation

Consider a network consisting of three nodes and a common destination as shown in Fig. 1. Fig. 2 shows the throughput of node 1 given variations in the average SNR from nodes to its destination. In Fig. 2, we assume that the average SNR of all nodes are identical for simplicity of presenting the result. We consider three different coalitions for node 1 (i.e., $\{1\}$, $\{1,2\}$, and $\{1,2,3\}$) in order to investigate the impact of relay transmission from the other nodes. It is observed that, at different SNRs, node 1 obtains its highest throughput by being a member of different coalitions. For instance, when the average SNR is small, the throughput without cooperation is the lowest due to poor channel quality. Therefore, node 1 finds it beneficial to cooperate by forming coalition with other nodes. In contrast, when the average SNR is high, node 1 gains a higher throughput without relay transmission. In this case, forming coalitions will degrade the performance since node 1 has to dedicate some resources (e.g., time slots) to perform relay transmission for the other nodes in the same coalition. Based on this observation, we can clearly see that coalition formation is a central issue for deciding on whether to perform cooperative transmission or not. In addition, this result shows how, in many scenarios, coalition formation can significantly improve the performance of the network nodes, in terms of throughput.

![Fig. 2](image-url)  
Throughput under different SNR from nodes to destination with different coalitions.

![Fig. 3](image-url)  
(a) Stable coalitions and (b) average throughput of nodes under varied SNR difference.

As the SNR difference between the nodes varies, Fig. 3(a) shows the stable coalitional structure resulting from the proposed coalition formation algorithm with a total of 4 nodes in the network. The SNR difference used in Fig. 3(a) is defined as $\Delta = \gamma_1 - \gamma_2 = \gamma_2 - \gamma_3 = \gamma_3 - \gamma_4$ where the average SNR of node 4 is fixed at $\gamma_4 = 4$ dB. That is, the average SNRs of nodes 1, 2, and 3 are varied. It is observed that, when the SNR difference is small, all nodes can gain a higher payoff (i.e., throughput) by forming a grand coalition (i.e., coalition of all nodes). Therefore, the grand coalition is stable. When the SNR difference becomes larger, the nodes can achieve a higher throughput by being forming smaller coalitions (i.e., being more non-cooperative). Therefore, the stable coalitions in this case have a smaller number of members (e.g., $\{2,3,4\}$ and $\{1,2,3\}$). When the SNR difference is very large (e.g., 2.5 dB), the majority of the nodes would have an incentive to not form any coalition since the throughput of direct transmission is high. Moreover, there could be multiple stable coalitional
structures (e.g., when the SNR difference is 1 dB). In this case, when the nodes reach any of these stable coalitional structures, they will remain using these structures forever (assuming a static environment).

Fig. 3(b) shows the average throughput of the nodes in a stable coalitional structure. As the SNR difference increases, node 4 will be the first to split from its coalition as it has the lowest SNR, since the other nodes would have no incentive to cooperate with node 4. As a result, the throughput of node 4 decreases first. Then, due to similar reasons, node 3 will be split from the coalition of nodes 1 and 2. As a result, throughput of node 3 decreases. Finally, nodes 2 will be split. Note that the split nodes can form their own coalition (e.g., \{3, 4\}) to improve their throughput. In addition, we can see from Fig. 3(b) that node 1 has the highest throughput even without belonging to a coalition. In particular, node 1 direct transmission throughput can be higher than its throughput when using relay transmission. This result is due to the fact that, when forming coalitions, node 1 has to use its allocated resource (i.e., time slot) to perform relay transmission for the other nodes. As a result, we can see that, under different conditions (e.g., channel quality), forming a coalition may or may not lead to the best performance for the nodes. This result motivates further the stability analysis of coalition formation that is performed in this paper.

<table>
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<th>Scenario 1 Lower bound</th>
<th>Scenario 1 Upper bound</th>
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Table I shows the upper and lower bounds probabilities of stable coalitional formation under an interval uncertainty for three nodes with relay transmission. The average SNR of nodes 1 and 2 are $\tau_1 = \tau_2 = 5$ dB. We consider two scenarios in which the range for the average SNR of node 3 is varied, i.e., $\tau_3^- = 4$ dB and $\tau_3^+ = 6$ dB for Scenario 1 and $\tau_3^+ = 2$ dB and $\tau_3^- = 6$ dB for Scenario 2. As shown in Table I, for Scenario 1, only the grand coalition $\{1, 2, 3\}$ is always stable (i.e., upper and lower bound probabilities are one) if the average SNR of node 3 falls into the range of $[4, 6]$ dB. In contrast, for Scenario 2, if the average SNR of node 3 falls into the range of $[3, 6]$ dB, only partitions $\{1, 2, 3\}$ and $\{1, 2\}, \{3\}$ can be stable in some cases due to the uncertainty of the average SNR. However, coalitional structures $\{1\}, \{2\}, \{3\}$, $\{1, 3\}, \{2\}$, and $\{1\}, \{2, 3\}$ will never be stable.

VI. SUMMARY

In a cooperative network, the wireless nodes can cooperate by forming coalitions in order to improve their throughput through relay transmission. In this paper, we have presented a coalitional game formulation for relay transmission when the nodes are rational and seek to maximize their individual throughput. In this game, the nodes can take a decision to join or split from a coalition, while taking into account the performance improvement, in terms of throughput, resulting from the coalition formation decision. For analyzing the stability of the proposed coalition formation game, we have proposed and studied a Markov chain-based model. In addition, this paper tackles the case in which the wireless nodes are uncertain about the parameters that will be used to perform coalition formation. Under this certainty, we have analyzed the stability of the coalition formation process using the lower and upper bounds of the stationary probability of the corresponding Markov chain.

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