Line Detection Filters

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SUMMARY In this paper, two new simple line detection filters have been designed and compared with two existing line detection filters, the Petrou filter and the Sombrero filter. The performances of these four filters are discussed both theoretically and experimentally.

1 INTRODUCTION

It is common knowledge that edge and line detection is one of the most important steps in the early stages of vision in image processing. There have been sporadic publications in the area of edge detection [1–9]. Canny [2–3] considered first the one-dimensional case edge detection problem in terms of an optimal edge detector and set the foundations of the theory of an optimal edge detector for a traditional model in the presence of white gaussian noise. A lengthy mathematical argument showed that his best optimal edge detecting operator (for application to the intensity profile across a step edge in the presence of white gaussian noise) could be closely approximated by the first derivative of the gaussian filter. Based on Canny’s criteria, Spacoe [8–9] simplified the process to obtain an optimal edge detection filter and a cubic spline filter. Later Petrou and Kittler [6] extended Spacoe’s work by applying it to ramp edges and performing the optimisation process exactly. The performance measures for a step edge show that the Petrou filter is superior among these filters and the cubic filter is a “good” suboptimal filter. Besides these works, several authors [1,4,5,7] have used different approaches to improve their optimal filters.

Line detection is a useful means of identifying wide linear features such as roads, hedges, canals and rivers in images or in digitised maps and to distinguish them from other types of image features. The line signal has the form of the first derivative of the step signal. Since lines and edges are related by spatial derivatives, so are their respective filters. The filter for localisation of an edge is the detection filter for a line. Utilising this relationship, some authors [1–3, 10–13] designed various filters based on edge detection theory. Unfortunately, the practical line detection filters in the published literature lack effectiveness. Two practical filters have been implemented for the one-dimensional case line detection problem. One is the Petrou filter [11] and another is the Sombrero filter, which is the second derivative of the gaussian filter.

An optimum line detection filter, \( f(x) \), of finite extent, 2\( \omega \), for an intensity profile, \( c(x) \), across a linear feature in the presence of noise should possess the following attributes:

a) **Good Detection** this corresponds to maximising the output signal-to-noise ratio from the line detection filter, ie, the following quantity is maximised:

\[
S = \frac{\int_{-\omega}^{\omega} f(x)c(-x)dx}{\sqrt{\int_{-\omega}^{\omega} |f(x)|^2 dx}}
\]  

b) **Good Localisation** this is equivalent to maximising the output signal slope-to-noise ratio from an optimum line location filter. It has been proven that the optimum line location filter is the first derivative of the optimum line detection filter and the following quantity should be maximised:

\[
L = \frac{\int_{-\omega}^{\omega} f(x)c'(-x)dx}{\sqrt{\int_{-\omega}^{\omega} |f'(x)|^2 dx}}
\]

c) **Minimal Number of False Responses** this means that the number of responses to a single line in its vicinity of the line must be minimised. Such an optimum line detection filter should be chosen so that it maximises the quantity:
\[ R = \frac{1}{\omega} \sqrt{\int_{-\infty}^{\infty} |f'(x)|^2 \, dx} \sqrt{\int_{-\infty}^{\infty} |f''(x)|^2 \, dx} \]  

where \( H(x) \) is the unit-step function, usually defined by:

\[
H(x) = \begin{cases} 
0, & \text{if } x < 0 \\
\frac{1}{\omega}, & \text{if } x = 0 \\
1, & \text{if } x > 0 
\end{cases}
\]

The half-width of the filter, \( \omega \), is greater than the half-width, \( r \), of the feature, i.e., \( \omega > r \). The scaled operator has been defined as:

\[
g(y) = f(x), \quad y = \frac{x}{\omega}
\]

The measure quantities are then given by:

\[
S = \sqrt{2} \left[ \int_{-\omega r/\omega}^{0} g(y) \, dy \right] \sqrt{\int_{-1}^{0} g^2(y) \, dy}
\]

\[
L = \sqrt{2} \left[ \int_{-\omega r/\omega}^{0} g'(y) \, dy \right] \sqrt{\int_{-1}^{0} g'^2(y) \, dy}
\]

\[
R = \sqrt{\int_{-1}^{0} g^2(y) \, dy} \sqrt{\int_{-1}^{0} g'^2(y) \, dy}
\]

The constraints (4) become:

\[
g(-1) = 0 \\
g'(-1) = 0 \\
g(0) = k \\
g'(0) = 0 \\
\int_{-1}^{0} g(y) \, dy = 0
\]

When the optimality measure is \( \frac{1}{2} L^2 R^2 \), the problem of optimal filter design is to choose a function, \( f(x) \) or \( g(y) \), which satisfies the boundary conditions (9) in order to maximise the measure:

A one-dimensional convolution filter, \( f(x) \), is required, which will detect lines and must have the following properties:

i) \( f(x) \) is a real symmetric (even) filter for detection of symmetric features, like lines, with intensity profiles described by a real even function, \( c(x) \).

ii) \( f(x) \) must be subject to the following constraints:

\[
f(-\omega) = 0
\]

\[
f'(\omega) = 0
\]

\[
f(0) = k
\]

\[
f'(0) = 0
\]

\[
\int_{-\omega}^{\omega} f(x) \, dx = 0
\]

The first and second constraint ensure that the finite extent goes smoothly to zero and avoids the consequences of Green's effect. The third and fourth constraints guarantee that the maximum given amplitude, \( k \), is reached at the origin and that the symmetric constraint can also be satisfied. The final constraint ensures that the filter will not respond to a uniform signal.

It is known that it is impossible to choose a convolution filter which will maximise all three of these quantities. However, it is possible to combine them in some way, e.g., \( \frac{1}{2} L^2 R^2, \frac{1}{2} S L^2 R^2 = P \), SL or R, to find a solution which will maximise this measure of combined optimality.

This paper presents two new line detection filters and describes an investigation carried out to compare their performance with two existing line detection filters based on Canny's criteria.

2 QUARTIC FILTER

It is assumed that the idealised profile of \( c(x) \) to detect is:

\[
c(x) = H(r + x)H(r - x) = \begin{cases} 
0, & \text{if } x > r \\
\frac{1}{2}, & \text{if } x = r \\
1, & \text{if } -r < x < r \\
\frac{1}{2}, & \text{if } x = -r \\
0, & \text{if } x < -r 
\end{cases}
\]
\[ \frac{1}{2} L^2 R^2 = \frac{g'(\frac{r}{\omega})}{\int_{-1}^{0} g''(y)dy} \]

Note that the \[\int_{-1}^{0} g''(y)dy\] has been eliminated in the product LR.

Using variational methods \[\int_{-1}^{0} g''(y)dy\] is minimised. The variational function is defined as \[G(g,g''') = g^2 + \lambda g\]
where \(\lambda\) is a Lagrange multiplier. The Euler equation then becomes \[g''' = -\frac{\lambda}{2}\] and the solution is a quartic polynomial:

\[g(y) = -k(15y^4 - 32y^2 + 28y^2 - 1)\]

\[= -k((3|y| - 1)^2 (3|y| - 1)(5|y| + 1)\]

where \(\lambda = 720k\). Substitution of the expression into \(\frac{1}{2} L^2 R^2\) yields the following result:

\[\frac{1}{2} L^2 R^2 = \frac{75(\frac{r}{\omega})^6 - 240(\frac{r}{\omega})^5 + 282(\frac{r}{\omega})^4 - 144(\frac{r}{\omega})^3 + 27(\frac{r}{\omega})^2}{4}\]

This analysis gives the equation:

\[75(\frac{r}{\omega})^5 - 200(\frac{r}{\omega})^4 + 188(\frac{r}{\omega})^3 - 72(\frac{r}{\omega})^2 + 9(\frac{r}{\omega}) = 0\]

from which \(\omega\) can be found in terms of \(r\) to maximise the quantity \(\frac{1}{2} L^2 R^2\). The five solutions are \(\omega = r, \omega = 1.2137r, \omega = \frac{2}{3}r = 1.6667r, \omega = 4.1196r\) and \(\omega = +\infty\). The five values of:

\[\frac{1}{2k^2} S^2 \int_{-1}^{0} g''(y)dy = \frac{r}{\omega}(1 - \frac{r}{\omega})^3 (1 + \frac{3r}{\omega})\]

(has a maximum value of 0.1975 when \(\omega = 3r\)) are 0, 0.0156, 0.10175, 0.1822 and 0. The quartic filter should be in the form:

\[f(x) = -k(15y^4 - 32y^2 + 28y^2 - 1), \quad (10)\]

with

\[y = x/\omega, \quad \omega = 4.1196r\]

Its first derivative zero crossing is \(\omega = \frac{2}{3}r = 1.6667r\). The performance values are:

\[S = \frac{\sqrt{35}}{2} |3y^5 - 8y^4 + 6y^3 - y|_{y=r/\omega} = 0.5389\]

\[L = \frac{\sqrt{210}}{2} |5y^3 - 8y^2 + 3y|_{y=r/\omega} = 2.3791\]

\[R = \frac{1}{35} \sqrt{35} = 0.1690\]

\[SL = 1.2820\]

\[P = 0.0117\]

### 3 BAR FILTER

Now the following quantity is used as an optimality measure

\[P = \frac{1}{4} S^2 L^2 R^2 = \frac{\left[ g'((\frac{r}{\omega})^0) \right] \int_{-1}^{0} g(y)dy }{\int_{-1}^{0} g''(y)dy \int_{-1}^{0} g''(y)dy}\]

In order to choose a function which maximises the above performance measure, \(P\), one way is to maximise or minimise any one of the integrals appearing in its expression, by assuming that the remaining integrals are constant. \(\int_{-1}^{0} g''(y)dy\) is chosen for minimisation. The variational function is

\[G(g,g''') = g^2 + \lambda_1 g''' + \lambda_2 g\]

where \(\lambda_1\) and \(\lambda_2\) are two Lagrange multipliers. The Euler equation then becomes

\[2\lambda_1 g''' + 2g + \lambda_2 = 0\]

The general solution is as follows:

\[g(y) = K_1 \sin(Ay) \exp(Ay)\]

\[+ K_2 \cos(Ay) \exp(Ay)\]

\[+ K_3 \sin(Ay) \exp(-Ay)\]

\[+ K_4 \cos(Ay) \exp(-Ay) - \frac{\lambda_2}{2}\]

where \(A = \sqrt{2/2\lambda_1^{1/4}}\) and \(K_1, K_2, K_3\) and \(K_4\) are arbitrary constants. From the constraints (9) an analytical solution is obtained in terms of \(A\) and \(\lambda_2\)
\[
K_1 = \frac{\lambda_2 \exp(A)A \exp(3A) - \exp(3A) - \exp(2A) \sin(A)}{2 \left( \exp(4A) - 2 \exp(3A) \sin(A) \right) - 2 \exp(3A) \cos(A) + 2 \exp(2A) \sin(2A) - 2 \exp(A) \sin(A) + 2 \exp(A) \cos(A) - 1} \\
K_2 = \frac{\lambda_2 \exp(A)A \exp(3A) - \exp(3A) + \exp(2A) \sin(A)}{2 \left( \exp(4A) - 2 \exp(3A) \sin(A) \right) - 2 \exp(3A) \cos(A) + 2 \exp(2A) \sin(2A) - 2 \exp(A) \sin(A) + 2 \exp(A) \cos(A) - 1} \\
K_3 = \frac{\lambda_2 \exp(A)A \exp(3A) - \exp(3A) \cos(A)}{2 \left( \exp(4A) - 2 \exp(3A) \sin(A) \right) - 2 \exp(3A) \cos(A) + 2 \exp(2A) \sin(2A) - 2 \exp(A) \sin(A) + 2 \exp(A) \cos(A) - 1} \\
K_4 = \frac{\lambda_2 \exp(A)A \exp(3A) + \exp(3A) \cos(A)}{2 \left( \exp(4A) - 2 \exp(3A) \sin(A) \right) - 2 \exp(3A) \cos(A) + 2 \exp(2A) \sin(2A) - 2 \exp(A) \sin(A) + 2 \exp(A) \cos(A) - 1} 
\]

\[
K_1 = 0.3672 \\
K_2 = -0.7728 \\
K_3 = -0.1674 \\
K_4 = -0.2382 \\
\lambda_2 = 2(K_2 + K_4 - k) 
\]

When \( \omega = 3.5483r \) is chosen, the performance measure \( P \) can obtain its maximum value. The performance values are \( S = 0.5697, L = 2.3330, R = 0.1690, SL = 1.3291, \) and \( P = 0.0126. \)

4 PETROU FILTER

Petrou [11] has described an optimal filter for characterising ridges. The parameters of the Petrou-kernel are predetermined by optimising Canny's criteria but embodying the scale dependent features of ridges. Petrou assumes the following mathematical description for the model:

\[
c(x) = \begin{cases} 
A_1 \exp(-x) - A_2 \exp(-sx), & \text{if } x > d \\
A_3 \cosh(sx) - A_4 \cosh(x) + A_5, & \text{if } d \leq x \leq -d \\
A_1 \exp(x) - A_2 \exp(sx), & \text{if } x < -d 
\end{cases} 
\]

The parameters are given by:

\[
A_1 = A_0 s^2 \sinh(ld) \\
A_2 = A_0 s^2 \sinh(sd) \\
A_3 = A_0 d^2 \exp(-ld) \\
A_4 = A_0 s^2 \exp(-sd) \\
A_5 = A_0(s^2-l^2) 
\]

where

\[
A_0 = \frac{1}{s^2[1 - \exp(-ld)] - l^2[1 - \exp(-sd)]} 
\]
The amplitude factor of this model has been modified so that e(0) = 1. The calculus of variations is used in exactly the same manner as in the previous section in order to maximise the following performance measure P:

\[
P = \frac{1}{4} s^2 L^2 R^2 \left[ \int_0^1 g(y)e(s\omega)dy \int_0^1 g''(y)c(s\omega)dy \right]^2
\]

(20)

Hence the composite functional becomes

\[
G(g,g'') = g^2(y) + \lambda_2 g^2(y) + \lambda_2 g(y) + \lambda_3 g(y)c(s\omega) + \lambda_4 g''(y)c(s\omega)
\]

The Euler equation becomes:

\[
2g(y) + 2\lambda_4 g''(y) = -\lambda_2 - \lambda_3 c(s\omega) - \lambda_4 s^2 c''(s\omega)
\]

The real solution for the optimal problem is given by the following:

for \(-d/\omega < y < -d/\omega\)

\[
g(y) = K_1 e^{\omega y} \cos(s\omega) + K_2 e^{\omega y} \sin(s\omega) + K_3 e^{-\omega y} \cos(s\omega) + K_4 e^{-\omega y} \sin(s\omega) + A_1 K_5 e^{s\omega y} - A_2 K_6 e^{s\omega y} + K_7
\]

for \(-d/\omega \leq y \leq 0\)

\[
g(y) = N_1 e^{\omega y} \cos(s\omega) + N_2 e^{\omega y} \sin(s\omega) + N_3 e^{-\omega y} \cos(s\omega) + N_4 e^{-\omega y} \sin(s\omega) + A_3 K_6 \cosh(sy) - A_4 K_5 \cosh(sy) + N_5 + K_7
\]

where

\[
\alpha = \frac{\sqrt{2}}{2\lambda_1^4}
\]

\[
K_5 = \frac{\lambda_3 + \lambda_4 l^2 \omega^2}{2(1 + \lambda_1 l^4 \omega^4)}
\]

\[
K_6 = \frac{\lambda_3 + \lambda_4 s^2 \omega^2}{2(1 + \lambda_1 s^4 \omega^4)}
\]

\[
K_7 = \frac{-\lambda_2}{2}
\]

\[
N_5 = A_0 \left[ s^2 \left( 1 + \frac{l^4 \omega^4}{4\alpha^4} \right) K_5 - l^2 \left( 1 + \frac{s^4 \omega^4}{4\alpha^4} \right) K_6 \right]
\]

Petrou simply sets the constant deviation, \(K_7\), equal to zero. The parameters of the Petrou filter when \(d = 1\) and \(l = 10\) are shown in Table 1 and the performance values are \(S = 0.7721\), \(L = 2.1205\), \(R = 0.1651\), \(SL = 1.6373\), and \(P = 0.0183\).

<table>
<thead>
<tr>
<th>s</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_1</td>
<td>-88.3436</td>
<td>-99.2921</td>
<td>-121.8239</td>
<td>-139.6549</td>
<td>-148.5791</td>
<td>-314.9304</td>
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<tr>
<td>K_2</td>
<td>179.8679</td>
<td>196.1246</td>
<td>229.5944</td>
<td>256.1866</td>
<td>269.0995</td>
<td>499.5851</td>
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<td>K_3</td>
<td>92.0922</td>
<td>102.9407</td>
<td>125.4234</td>
<td>143.2741</td>
<td>152.1857</td>
<td>318.5423</td>
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<tr>
<td>K_4</td>
<td>43.1763</td>
<td>49.8561</td>
<td>64.0229</td>
<td>75.3821</td>
<td>81.2109</td>
<td>196.8533</td>
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<td>K_5</td>
<td>-0.0316</td>
<td>-0.0396</td>
<td>-0.0433</td>
<td>-0.0460</td>
<td>-0.0471</td>
<td>-0.0479</td>
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<td>K_6</td>
<td>-0.8092</td>
<td>-0.4469</td>
<td>-0.2737</td>
<td>-0.1853</td>
<td>-0.1313</td>
<td>-0.0980</td>
</tr>
<tr>
<td>N_1</td>
<td>684.7790</td>
<td>823.5345</td>
<td>1107.5242</td>
<td>1330.2933</td>
<td>1472.7771</td>
<td>3859.4010</td>
</tr>
<tr>
<td>N_3</td>
<td>756.9835</td>
<td>877.7348</td>
<td>1152.1922</td>
<td>1367.9227</td>
<td>1501.1228</td>
<td>3903.4828</td>
</tr>
<tr>
<td>N_4</td>
<td>90</td>
<td>69</td>
<td>63</td>
<td>59</td>
<td>54</td>
<td>84</td>
</tr>
<tr>
<td>N_5</td>
<td>-1440.8952</td>
<td>-1700.2952</td>
<td>-2258.7225</td>
<td>-2697.2177</td>
<td>-2972.9003</td>
<td>-7761.8839</td>
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<tr>
<td>(\alpha)</td>
<td>0.4338</td>
<td>0.42</td>
<td>0.395</td>
<td>0.38</td>
<td>0.373</td>
<td>0.2944</td>
</tr>
<tr>
<td>(\omega)</td>
<td>4.82</td>
<td>4.2</td>
<td>3.95</td>
<td>3.8</td>
<td>3.73</td>
<td>3.68</td>
</tr>
</tbody>
</table>
5 SOMBRERO FILTER (MEXICAN HAT FILTER)

From signal processing theory, it is known that in general a filter with a large window size will demonstrate better performance for reducing random noise. Nevertheless, the limited window size filter often introduces a cut-off effect in its derivatives at the window boundary. If the window size is not limited, an infinite extent symmetric filter should be used. The boundary conditions, given by (4), for a finite extent filter defined in the range \([-\omega, \omega]\), change as follows:

\[
f(-\infty) = 0
\]
\[
f'(0) = \frac{k}{\omega}
\]
\[
f'(0) = 0
\]
\[
\int_{-\infty}^{0} f(x)dx = 0
\]

A reasonable approximation to the optimal operator which satisfies the above boundary conditions is the second derivative of the gaussian filter:

\[
f(x) = k \left( 1 - \frac{x^2}{\sigma^2} \right) \exp\left( -\frac{x^2}{2\sigma^2} \right)
\]

This filter is called Sombrero filter or Mexican hat filter due to its shape. \(\sigma\) is a standard deviation of the gaussian filter. Its first derivative zero crossing is \(\sqrt{3}\sigma\). It is not possible to compare directly the performance of the Sombrero filter with that of other finite width filters. This is because the Sombrero filter does not satisfy the same boundary conditions as other finite width filters. The infinite convolution integrals provide an upper limit on the actual performance attainable by a real truncated Sombrero filter. The results of the infinite integrals will be expressed in terms of \(\alpha\), as yet undetermined, standard deviation, \(\sigma\), of the Sombrero filter. A particular value of \(\sigma\) is selected so as to obtain a numerical value for our performance measure. This step is accomplished by relating \(\sigma\) to the actual width of a truncated filter and this must be done before such a filter can be implemented. The Sombrero filter is truncated at a point \(\omega\), where the value of the Sombrero filter falls to about 0.001 of its peak value. Such a small value is intended to ensure that the real performance will not be much less than the theoretical upper limit obtained for the untruncated Sombrero filter. This determines the value of \(\sigma = 0.3189\omega\).

The integrals (1) to (3) are evaluated from limits \(-\infty\) to zero, as if no truncation took place.

\[
\begin{align*}
S &= \frac{4\pi^3 \exp\left( -\frac{r^2}{2\sigma^2} \right)}{3/\sigma \omega^{3/2}} \\
L &= \frac{4\pi^3 30\sigma \omega \left( 3 - \frac{r^2}{\sigma^2} \right) \exp\left( -\frac{r^2}{2\sigma^2} \right)}{15^{3/2} \pi \sigma^2} \\
R &= \frac{\alpha / 14}{7\omega} \\
SL &= \frac{16\pi^2}{10} \left( 3 - \frac{r^2}{\sigma^2} \right) \exp\left( -\frac{r^2}{\sigma^2} \right) \\
P &= \frac{25 \pi^4}{351 \pi^2 \omega^2} \left( 3 - \frac{r^2}{\sigma^2} \right)^2 \exp\left( -\frac{2r^2}{\sigma^2} \right)
\end{align*}
\]

When \(r = 1\) and \(\omega = 4.1196\), the performance values are \(S = 0.5581\), \(L = 2.6792\), \(R = 0.1705\), \(SL = 1.4953\) and \(P = 0.0162\).

6 EXPERIMENTAL RESULTS AND CONCLUSION

Now that four line detection filters have been described in detail, it is time to turn to a broader question: What is the best possible filter for line detection? To answer this question, both quantitative and experimental comparisons should be made among these filters.

For quantitative comparison, Canny's performance measures can be considered as a quantitative criteria. These performance values of various filters discussed are given in table 2. In reality, image lines are never ideal combined steps. Even if scene lines had idealised mathematical shapes to begin with, during the process of image capture and digitisation, the combined steps will be converted into combined ramps. For this reason, the Petrou filter seems to show the best performance according to the combined performance measure, \(P\). The signal-to-noise ratio is particularly good but this has been achieved at the expense of localisation. It is important to note that the procedure of calculating the parameters of the Petrou filter is too complicated so that it is computationally expensive to recalculate the Petrou filter parameter for a given structure of line features. One way around this problem could be to list some typical filter parameters and make some rules to build a best approximation to the desired optimal filter. Objectively the optimisation is restricted. The situation is analogous to those proposed by

Table 2
Performance measures of various filters for line detection

<table>
<thead>
<tr>
<th>Filter</th>
<th>S</th>
<th>L</th>
<th>R</th>
<th>S*L</th>
<th>L*R</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartic</td>
<td>0.5389</td>
<td>2.3791</td>
<td>0.1690</td>
<td>1.2820</td>
<td>0.4021</td>
<td>0.0117</td>
</tr>
<tr>
<td>Bar</td>
<td>0.5697</td>
<td>2.3330</td>
<td>0.1690</td>
<td>1.3291</td>
<td>0.3943</td>
<td>0.0126</td>
</tr>
<tr>
<td>Petrou</td>
<td>0.7721</td>
<td>2.1205</td>
<td>0.1651</td>
<td>1.6373</td>
<td>0.3501</td>
<td>0.0183</td>
</tr>
<tr>
<td>Sombrero</td>
<td>0.5581</td>
<td>2.6792</td>
<td>0.1705</td>
<td>1.4953</td>
<td>0.4568</td>
<td>0.0162</td>
</tr>
</tbody>
</table>

Table 3
Summary of line detection filters

<table>
<thead>
<tr>
<th>Line detectors</th>
<th>Remarks</th>
<th>Corresponding edge detectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartic</td>
<td>Simplest, fastest, handiest</td>
<td>Cubic [8,9]</td>
</tr>
<tr>
<td>Bar</td>
<td>Optimal for combined steps</td>
<td>Spacek [8,9]</td>
</tr>
<tr>
<td>Petrou</td>
<td>Optimal for combined ramps</td>
<td>Petrou [6]</td>
</tr>
<tr>
<td>Sombrero</td>
<td>Easiest for 2D extension</td>
<td>Canny [2,3]</td>
</tr>
</tbody>
</table>

Figure 1 Profiles of various line detection filters.

Figure 2 An original test image.

Canny for step edge detection. The new filters designed here, the Quartic and the Bar filter, are much simpler to use; no parameter need be chosen except the size of window. The reason for employing the Sombrero filter in this way is that there are very efficient means of computing the two-dimensional extension of the filter if it can be represented as some standard deviation of the gaussian filter. The shapes for various filters are shown in fig. 1. The kernels of these filters have similarity.

For experimental comparison, it is important to compare the various filters using a natural image. For this reason, the domain of interest is that of automated vehicle guidance (AVG) of airborne vehicles. A typical test image has been taken from infra-red linescan image of the Luton (England) area from a flight at 4000 feet. The full frame image is 512 x 512 pixels. Figure 2 shows the original image. The lines to recover and the results of running a line detection program using the Quartic filter (10), the Bar filter (13), the Petrou filter (21) and the Sombrero filter (23) are shown in fig. 3. All filters were implemented in the same algorithm which was coded in FORTRAN and run on Sun Sparc station 1+ computer under UNIX. The outputs are combined filter outputs in horizontal and vertical directions. It must be stressed here that no postprocessing procedures, eg, hysteresis thresholding, a thinning process of lines, or the removal of small isolated line segments, were used. There is almost no visible difference among these filters due to the similarity of the filter shapes.

It is worth noting that a high performance measure theoretically does not necessarily mean good performance.
in practice. Because Canny's performance measure is based on idealised mathematical line shapes immersed in perfect white gaussian noise, image analysis indicates that this assumption is often not true in practice. Table 3 shows a brief summary of line detection filters and corresponding edge detection filters.

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8 REFERENCES


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