Problem Definitions and Evaluation Criteria for the CEC 2010 Competition on Constrained Real-Parameter Optimization

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Introduction

Most real world optimization problems have constraints of different types (e.g., physical, time, geometric, etc.) which modify the shape of the search space. During the last couple of decades, a wide variety of metaheuristics have been designed and applied to solve constrained optimization problems [1]. Evolutionary algorithms and most other metaheuristics, when used for optimization, naturally operate as unconstrained search techniques. Therefore, they require an additional mechanism to incorporate constraints into their fitness function.

Historically, the most common approach to incorporate constraints (both in evolutionary algorithms and in mathematical programming) is the penalty functions, which were originally proposed in the 1940s and later expanded by many researchers. Penalty functions have, in general, several limitations. Particularly, they are not a very good choice when trying to solve problem in which the optimum is on the boundary between the feasible and the infeasible regions or when the feasible region is disjoint. Additionally, penalty functions require a careful fine-tuning to determine the most appropriate penalty factors to be used with our metaheuristics. Researchers have also proposed a number of other approaches to handle constraints such as the self-adaptive penalty, epsilon constraint handling and stochastic ranking. Additionally, the analysis of the role of the search engine has also become an interesting research topic in the last few years. For example, evolution strategies (ES), evolutionary programming (EP), differential evolution (DE) and particle swarm optimization (PSO) have been found advantageous by some researchers over other metaheuristics such as the binary genetic algorithms (GA).

In CEC06 [2], 24 benchmark functions have been presented which have 2-20 dimensions and are not easily scalable. In addition, CEC 2006 benchmark has been solved satisfactorily by several methods. Therefore, it has become impossible to demonstrate the superior performance of newly designed algorithms. CEC05 [3] presents some of the scalable bound constrained problems. In [4] author proposed a test-case generator for constrained parameter optimization problems. In [5] the authors generated some scalable constrained problems. In this report, we present 18 benchmark functions which are scalable. The mathematical formulas and properties of these functions are described in Section 1. In Section 2, the evaluation criteria are given. A suggested format to present the results is given in Section 3.

1. Definitions of the Function Suite

In this section, 18 optimization problems with constraints are described. They are all transformed into the following format:

Minimize: \( f(X) \), \( X = (x_1, x_2, ..., x_n) \) and \( X \in S \)  \( \ldots (1) \)

subject to:
\[
\begin{align*}
g_i(X) & \leq 0, & i = 1, ..., p \\
h_j(X) & = 0, & j = p + 1, ..., m 
\end{align*}
\]  \( \ldots (2) \)

Usually equality constraints are transformed into inequalities of the form
\[
|h_j(X)| - \varepsilon \leq 0, \text{ for } j = p + 1, ..., m \quad \ldots (3)
\]

A solution \( X \) is regarded as feasible if \( g_i(X) \leq 0 \), for \( i = 1, ..., p \) and \( |h_j(X)| - \varepsilon \leq 0 \), for \( j = p + 1, ..., m \). In this special session \( \varepsilon \) is set to 0.0001.
A constrained problem, in which the feasible patches are parallel to the axes (Figure 1), can be solved better by algorithms employing line search or difference of two or more solution vectors (such as DE). Therefore, to avoid the test problems from being biased to a particular class of algorithms, we rotate the constraints in most of the test problems. The effect of constraints can be observed in Figures 1 & 2. In Figure 2, it can be observed that the points A, B, C, and D have been rotated in the clockwise direction.
\[
\text{C01} \quad \text{Min } f(x) = \left| \frac{D}{\prod_{i=1}^{D} \cos^4(z_j) - 2D \prod_{i=1}^{D} \cos^2(z_j)} \sqrt{\sum_{i=1}^{D} |z_i|^2} \right| \quad z = x - o
\]

\[
g_1(x) = 0.75 - \prod_{i=1}^{D} z_i \leq 0
\]

\[
g_2(x) = \sum_{i=1}^{D} z_i - 7.5D \leq 0
\]

\[
x \in [0,10]^D
\]

\[
\text{C02} \quad \text{Min } f(x) = \max(z) \quad z = x - o, \quad y = z - 0.5
\]

\[
g_1(x) = 10 - \frac{1}{D} \sum_{i=1}^{D} \left[ z_i^2 - 10 \cos(2\pi z_i) + 10 \right] \leq 0
\]

\[
g_2(x) = \frac{1}{D} \sum_{i=1}^{D} \left[ z_i^2 - 10 \cos(2\pi z_i) + 10 \right] - 15 \leq 0
\]

\[
h(x) = \frac{1}{D} \sum_{i=1}^{D} \left[ y_i^2 - 10 \cos(2\pi y_i) + 10 \right] - 20 = 0
\]

\[
x \in [-5.12, 5.12]^D
\]

\[
\text{C03} \quad \text{Min } f(x) = \sum_{i=1}^{D-1} \left( 100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2 \right) \quad z = x - o
\]

\[
h(x) = \sum_{i=1}^{D-1} (z_i - z_{i+1})^2 = 0
\]

\[
x \in [-1000,1000]^D
\]

\[
\text{C04} \quad \text{Min } f(x) = \max(z) \quad z = x - o
\]

\[
h_1(x) = \frac{1}{D} \sum_{i=1}^{D} (z_i \cos(\sqrt{|z_i|})) = 0
\]

\[
h_2(x) = \sum_{i=1}^{D^2/2-1} (z_i - z_{i+1})^2 = 0
\]

\[
h_3(x) = \sum_{i=D/2+1}^{D-1} (z_i^2 - z_{i+1})^2 = 0
\]

\[
h_4(x) = \sum_{i=1}^{D} z = 0
\]

\[
x \in [-50,50]^D
\]
C05 \[ \text{Min } f(x) = \max(z) \quad z = x - o \]
\[ h_i(x) = \frac{1}{D} \sum_{i=1}^{D} (-z_i \sin(\sqrt{|z_i|})) = 0 \]
\[ h_z(x) = \frac{1}{D} \sum_{i=1}^{D} (-z_i \cos(0.5 \sqrt{|z_i|})) = 0 \]
\[ x \in [-600, 600]^D \]

C06 \[ \text{Min } f(x) = \max(z) \]
\[ z = x - o, \quad y = (x + 483.6106156535 - o)M - 483.6106156535 \]
\[ h_i(x) = \frac{1}{D} \sum_{i=1}^{D} (-y_i \sin(\sqrt{|y_i|})) = 0 \]
\[ h_z(x) = \frac{1}{D} \sum_{i=1}^{D} (-y_i \cos(0.5 \sqrt{|y_i|})) = 0 \]
\[ x \in [-600, 600]^D \]

C07 \[ \text{Min } f(x) = \sum_{i=1}^{D-1} \left( 100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2 \right) \]
\[ z = x + 1 - o, \quad y = x - o \]
\[ g(x) = 0.5 - \exp(-0.1 \sqrt{\frac{1}{D} \sum_{i=1}^{D} y_i^2}) - 3 \exp(\frac{1}{D} \sum_{i=1}^{D} \cos(0.1y_i)) + \exp(1) \leq 0 \]
\[ x \in [-140, 140]^D \]

C08 \[ \text{Min } f(x) = \sum_{i=1}^{D-1} \left( 100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2 \right) \]
\[ z = x + 1 - o, \quad y = (x - o)M \]
\[ g(x) = 0.5 - \exp(-0.1 \sqrt{\frac{1}{D} \sum_{i=1}^{D} y_i^2}) - 3 \exp(\frac{1}{D} \sum_{i=1}^{D} \cos(0.1y_i)) + \exp(1) \leq 0 \]
\[ x \in [-140, 140]^D \]

C09 \[ \text{Min } f(x) = \sum_{i=1}^{D-1} \left( 100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2 \right) \]
\[ z = x + 1 - o, \quad y = x - o \]
\[ h(x) = \sum_{i=1}^{D} (y \sin(\sqrt{|y_i|})) = 0 \]
\[ x \in [-500, 500]^D \]
C10 \[ \text{Min } f(x) = \sum_{i=1}^{D-1} (100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2) \]
\[ z = x + 1 - o, \quad y = (x-o)M \]
\[ h(x) = \sum_{i=1}^{D} (y \sin(\sqrt{|y_i|})) = 0 \]
\[ x \in [-500, 500]^D \]

C11 \[ \text{Min } f(x) = \frac{1}{D} \sum_{i=1}^{D} (-z_i \cos(2\sqrt{|z_i|})) \]
\[ z = (x-o)M, \quad y = x + 1 - o \]
\[ h(x) = \sum_{i=1}^{D-1} (100(y_i^2 - y_{i+1})^2 + (y_i - 1)^2) = 0 \]
\[ x \in [-100, 100]^D \]

C12 \[ \text{Min } f(x) = \sum_{i=1}^{D} (z_i \sin(\sqrt{|z_i|})) \quad z = x - o \]
\[ h(x) = \sum_{i=1}^{D-1} (z_i^2 - z_{i+1})^2 = 0 \]
\[ g(x) = \sum_{i=1}^{D} (z - 100 \cos(0.1z) + 10) \leq 0 \]
\[ x \in [-1000, 1000]^D \]

C13 \[ \text{Min } f(x) = \frac{1}{D} \sum_{i=1}^{D} (-z_i \sin(\sqrt{|z_i|})) \quad z = x - o \]
\[ g_1(x) = -50 + \frac{1}{100D} \sum_{i=1}^{D} z_i^2 \leq 0 \]
\[ g_2(x) = \frac{50}{D} \sum_{i=1}^{D} \sin\left(\frac{1}{50} \pi x\right) \leq 0 \]
\[ g_3(x) = 75 - 50(\sum_{i=1}^{D} \frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1) \leq 0 \]
\[ x \in [-500, 500]^D \]
C14  \[\min f(x) = \sum_{i=1}^{D-1} (100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2)\]
\[z = x + 1 - o, y = x - o\]
\[g_1(x) = \sum_{i=1}^{D} (-y_i \cos(\sqrt{|y_i|})) - D \leq 0\]
\[g_2(x) = \sum_{i=1}^{D} (y_i \cos(\sqrt{|y_i|})) - D \leq 0\]
\[g_3(x) = \sum_{i=1}^{D} (y_i \sin(\sqrt{|y_i|})) - 10D \leq 0\]
\[x \in [-1000, 1000]^D\]

C15  \[\min f(x) = \sum_{i=1}^{D-1} (100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2)\]
\[z = x + 1 - o, y = (x - o)M\]
\[g_1(x) = \sum_{i=1}^{D} (-y_i \cos(\sqrt{|y_i|})) - D \leq 0\]
\[g_2(x) = \sum_{i=1}^{D} (y_i \cos(\sqrt{|y_i|})) - D \leq 0\]
\[g_3(x) = \sum_{i=1}^{D} (y_i \sin(\sqrt{|y_i|})) - 10D \leq 0\]
\[x \in [-1000, 1000]^D\]

C16  \[\min f(x) = \sum_{i=1}^{D} \left(\frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos(\frac{z_i}{\sqrt{i}}) + 1\right)\]
\[z = x - o\]
\[g_1(x) = \sum_{i=1}^{D} \left[z_i^2 - 100 \cos(\pi z_i) + 10\right] \leq 0\]
\[g_2(x) = \prod_{i=1}^{D} z_i \leq 0\]
\[h_i(x) = \sum_{i=1}^{D} (x_i \sin(\sqrt{|x_i|})) = 0\]
\[h_i(x) = \sum_{i=1}^{D} (-x_i \sin(\sqrt{|x_i|})) = 0\]
\[x \in [-10, 10]^D\]
\[ C17 \quad \text{Min } f(x) = \sum_{i=1}^{D-1} (z_i - z_{i+1})^2 \quad z = x - o \]
\[ g_1(x) = \prod_{i=1}^{D} z_i \leq 0 \]
\[ g_2(x) = \sum_{i=1}^{D} z_i \leq 0 \]
\[ h(x) = \sum_{i=1}^{D} (z_i \sin(4\sqrt{|z_i|})) = 0 \]
\[ x \in [-10, 10]^D \]

\[ C18 \quad \text{Min } f(x) = \sum_{i=1}^{D-1} (z_i - z_{i+1})^2 \quad z = x - o \]
\[ g(x) = \frac{1}{D} \sum_{i=1}^{D} (-z_i \sin(\sqrt{|z_i|})) \leq 0 \]
\[ h(x) = \frac{1}{D} \sum_{i=1}^{D} (z_i \sin(\sqrt{|z_i|})) = 0 \]
\[ x \in [-50, 50]^D \]

**Table 1:** Details of 18 test problems. \( D \) is the number of decision variables, \( \rho = |F|/|S| \) is the estimated ratio between the feasible region and the search space, \( I \) is the number of inequality constraints, \( E \) is the number of equality constraints.

<table>
<thead>
<tr>
<th>Problem/Search Range</th>
<th>Type of Objective</th>
<th>Number of Constraints</th>
<th>Feasibility Region (( \rho ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E ) ( I ) 10D 30D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C01 [0,10] ( D )</td>
<td>Non Separable</td>
<td>0</td>
<td>2 Non Separable</td>
</tr>
<tr>
<td>C02 [-5.12,5.12] ( D ) Separable</td>
<td>1 Separable</td>
<td>2 Separable</td>
<td>0.000000 0.000000</td>
</tr>
<tr>
<td>C03 [-1000,1000] ( D )</td>
<td>Non Separable</td>
<td>1 Non Separable</td>
<td>0</td>
</tr>
<tr>
<td>C04 [-50,50] ( D ) Separable</td>
<td>4</td>
<td>2 Non Separable, 2 Separable</td>
<td>0</td>
</tr>
<tr>
<td>C05 [-600,600] ( D )</td>
<td>Separable</td>
<td>2 Separable</td>
<td>0</td>
</tr>
<tr>
<td>C06 [-600,600] ( D )</td>
<td>Separable</td>
<td>2 Rotated</td>
<td>0</td>
</tr>
<tr>
<td>C07</td>
<td>[-140,140]^D</td>
<td>Non Separable</td>
<td>0</td>
</tr>
<tr>
<td>C08</td>
<td>[-140,140]^D</td>
<td>Non Separable</td>
<td>0</td>
</tr>
<tr>
<td>C09</td>
<td>[-500500]^D</td>
<td>Non Separable</td>
<td>1 Separable</td>
</tr>
<tr>
<td>C10</td>
<td>[-500,500]^D</td>
<td>Non Separable</td>
<td>1 Rotated</td>
</tr>
<tr>
<td>C11</td>
<td>[-100,100]^D</td>
<td>Rotated</td>
<td>1 Non Separable</td>
</tr>
<tr>
<td>C12</td>
<td>[-1000,1000]^D</td>
<td>Separable</td>
<td>1 Non Separable</td>
</tr>
<tr>
<td>C13</td>
<td>[-500,500]^D</td>
<td>Separable</td>
<td>0</td>
</tr>
<tr>
<td>C14</td>
<td>[-1000,1000]^D</td>
<td>Non Separable</td>
<td>0</td>
</tr>
<tr>
<td>C15</td>
<td>[-1000,1000]^D</td>
<td>Non Separable</td>
<td>0</td>
</tr>
<tr>
<td>C16</td>
<td>[-10,10]^D</td>
<td>Non Separable</td>
<td>2 Separable</td>
</tr>
<tr>
<td>C17</td>
<td>[-10,10]^D</td>
<td>Non Separable</td>
<td>1 Separable</td>
</tr>
<tr>
<td>C18</td>
<td>[-50,50]^D</td>
<td>Non Separable</td>
<td>1 Separable</td>
</tr>
</tbody>
</table>

2. **Performance Evaluation Criteria**

Number of Problems: 18. Number of runs/trials: 25

Maximum Function Evaluations (Max_FES) = 200000 for 10D and 600000 for 30D

Population Size: You are free to have an appropriate population size to suit your algorithm while not exceeding the Max FES.

2.1 **Presentation of Statistics**

Record the function value of \( f(X) \) for the achieved best solution \( X \) after 20000, 100000 and 200000 for 10D and 60000, 300000, 600000 for 30D. For each function, present the following: best, median, worst result, mean value and standard deviation for the 25 runs.
Please indicate the number of violated constraints (including the number of violations by more than 1, 0.01, and 0.0001) and the mean violations \( \bar{v} \) at the median solution.

\[
v = \frac{\left( \sum_{i=1}^{p} G_i(X) + \sum_{j=p+1}^{m} H_j(X) \right)}{m}
\]

where

\[
G_i(X) = \begin{cases} 
  g_i(X) & \text{if } g_i(X) > 0 \\
  0 & \text{if } g_i(X) \leq 0 
\end{cases}
\]

\[
H_j(X) = \begin{cases} 
  |h_j(X)| & \text{if } |h_j(X)| - \epsilon > 0 \\
  0 & \text{if } |h_j(X)| - \epsilon \leq 0 
\end{cases}
\]

2.2 Feasibility Rate
Feasible Run: A run during which at least one feasible solution is found in Max FES.
Feasible Rate = (# of feasible runs) / Total runs.
The above quantity is computed for each problem separately.

2.3 Algorithm Complexity
a) \( T_1 = \frac{\sum_{i=1}^{18} t1i}{18} \). \( t1i \) is the computing time of 10000 evaluations for problem \( i \).
b) \( T_2 = \frac{\sum_{i=1}^{18} t2i}{18} \). \( t2i \) is the complete computing time for the algorithm with 10000 evaluations for problem \( i \).
The complexity of the algorithm is reflected by: \( T_1 \); \( T_2 \); and \( (T_2-T_1)/T_1 \)

2.4 Parameters
We discourage participants searching for a distinct set of parameters for each problem/dimension/etc. Please provide details on the following whenever applicable:

a) All parameters to be adjusted.
b) Corresponding dynamic ranges.
c) Guidelines on how to adjust the parameters.
d) Estimated cost of parameter tuning in terms of number of FEs.
e) Actual parameter values used.

2.5 Encoding
If the algorithm requires encoding, then the encoding scheme should be independent of the specific problems and governed by generic factors such as the search ranges.

3. Presentation of Results
Participants are suggested to present their results in the following format:

**PC Configure:**

System:                                                CPU:        RAM:  
Language:                  Algorithm:  

**Parameters Setting:**

a) All parameters to be adjusted.  
b) Corresponding dynamic ranges.  
c) Guidelines on how to adjust the parameters.  
d) Estimated cost of parameter tuning in terms of number of FEs.  
e) Actual parameter values used.  

**Results Obtained**

Table 2: Function Values Achieved When FES = $2 \times 10^4$, $FES = 1 \times 10^5$, $FES = 2 \times 10^5$ for 10D Problems C01-C06.

<table>
<thead>
<tr>
<th>FEs</th>
<th>C01</th>
<th>C02</th>
<th>C03</th>
<th>C04</th>
<th>C05</th>
<th>C06</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 10^4$</td>
<td>Best: 237.9718</td>
<td>Median: 358.3837</td>
<td>worst: 446.8061</td>
<td>c: 2, 0, 0</td>
<td>v̄: 5.3256</td>
<td>Mean: 350.3861</td>
</tr>
<tr>
<td>$1 \times 10^5$</td>
<td>Best: 152.1540</td>
<td>Median: 291.1380</td>
<td>worst: 386.3278</td>
<td>c: 0, 2, 0</td>
<td>v̄: 4.12E-05</td>
<td>Mean: 28.1940</td>
</tr>
<tr>
<td>$2 \times 10^5$</td>
<td>Best: -158.7482</td>
<td>Median: -55.7482</td>
<td>worst: 38.5729</td>
<td>c: 0, 0, 0</td>
<td>v̄: 0</td>
<td>Mean: -69.0852</td>
</tr>
</tbody>
</table>

Table 3: Function Values Achieved When FES = $2 \times 10^4$, $FES = 1 \times 10^5$, $FES = 2 \times 10^5$ for 10D Problems C07-C12.

<table>
<thead>
<tr>
<th>FEs</th>
<th>C07</th>
<th>C08</th>
<th>C09</th>
<th>C10</th>
<th>C11</th>
<th>C12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 10^4$</td>
<td>Best</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEs</td>
<td>C13</td>
<td>C14</td>
<td>C15</td>
<td>C16</td>
<td>C17</td>
<td>C18</td>
</tr>
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<tr>
<td>$2 \times 10^4$</td>
<td>Best</td>
<td>Median</td>
<td>worst</td>
<td>$c$</td>
<td>$\bar{p}$</td>
<td>Mean</td>
</tr>
<tr>
<td>$1 \times 10^5$</td>
<td>Best</td>
<td>Median</td>
<td>worst</td>
<td>$c$</td>
<td>$\bar{p}$</td>
<td>Mean</td>
</tr>
<tr>
<td>$2 \times 10^5$</td>
<td>Best</td>
<td>Median</td>
<td>worst</td>
<td>$c$</td>
<td>$\bar{p}$</td>
<td>Mean</td>
</tr>
</tbody>
</table>

Table 4: Function Values Achieved When FES = $2 \times 10^4$, FES = $1 \times 10^5$, FES = $2 \times 10^5$ for Problems C13-C18 of 10D.
Table 5: Function Values Achieved When FES = $6 \times 10^4$, FES = $3 \times 10^5$, FES = $6 \times 10^5$ for Problems C01-C06 of 30D.

<table>
<thead>
<tr>
<th>FEs</th>
<th>C01</th>
<th>C02</th>
<th>C03</th>
<th>C04</th>
<th>C05</th>
<th>C06</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \times 10^4$</td>
<td>Best</td>
<td>Median</td>
<td>worst</td>
<td>c</td>
<td>p</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>std</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3 \times 10^5$</td>
<td>Best</td>
<td>Median</td>
<td>worst</td>
<td>c</td>
<td>p</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$6 \times 10^5$</td>
<td>Best</td>
<td>Median</td>
<td>worst</td>
<td>c</td>
<td>p</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td></td>
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</tbody>
</table>

Table 6: Function Values Achieved When FES = $6 \times 10^4$, FES = $3 \times 10^5$, FES = $6 \times 10^5$ for Problems C07-C12 of 30D.

<table>
<thead>
<tr>
<th>FEs</th>
<th>C07</th>
<th>C08</th>
<th>C09</th>
<th>C10</th>
<th>C11</th>
<th>C12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \times 10^4$</td>
<td>Best</td>
<td>Median</td>
<td>worst</td>
<td>c</td>
<td>p</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>std</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3 \times 10^5$</td>
<td>Best</td>
<td>Median</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Table 7: Function Values Achieved When FES = $6 \times 10^4$, FES = $3 \times 10^5$, FES = $6 \times 10^5$ for Problems C13-C18 of 30D.

<table>
<thead>
<tr>
<th>FEs</th>
<th>C13</th>
<th>C14</th>
<th>C15</th>
<th>C16</th>
<th>C17</th>
<th>C18</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \times 10^5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Best</td>
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<tr>
<td>Median</td>
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<tr>
<td>worst</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>std</td>
<td></td>
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</tbody>
</table>

$c$ is the number of violated constraints at the median solution: the sequence of three numbers indicate the number of violations (including inequality and equalities) by more than 1.0, more than 0.01 and more than 0.0001 respectively. $\bar{c}$ is the mean value of the violations of all
constraints at the median solution. The numbers in the parenthesis after the fitness value of
the best, median, worst solution are the number of constraints which cannot satisfy feasible
condition at the best, median and worst solutions respectively. Sorting method for the final
results:
1. Sort feasible solutions in front of infeasible solutions;
2. Sort feasible solutions according to their function values \( f(x^*) \)
3. Sort infeasible solutions according to their mean value of the violations of all constraints.

**Algorithm Complexity**

**Table 8: Computational Complexity**

<table>
<thead>
<tr>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>( (T_2-T_1)/T_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Convergence Graphs**

The participants are expected to plot the convergence plots for the 10D and 30D problems of
C09, C10, C14, C15, C17 and C18. The plot should show only feasible solutions of the best
run out of the 25 runs.

Plot 1 – Convergence plot for 10D problems C09, C10, C14 and C15.
Plot 2 – Convergence plot for 30D problems C09, C10, C14 and C15.
Plot 3 – Convergence plot for 10D problems C17 and C18.
Plot 4 – Convergence plot for 30D problems C17 and C18.

**Evaluation Criteria**

1. The algorithms should not use explicit equations. Only the use of function calls is
   allowed.
2. Gradients, etc. can only be computed numerically and the function evaluations consumed
   in the process of gradient computations should be accumulated.
3. Evaluation of even one constraint function should be treated as one function evaluation.

**References**


