Performance Analysis of Space Vector Modulation

Lim Shu Fan\textsuperscript{1}, Anshuman Tripathi\textsuperscript{2} and Ashwin M. Khambadkone\textsuperscript{3}

Department of Electrical and Computer Engineering, National University of Singapore

ABSTRACT

In this paper, an analytical method for calculating total harmonic distortion (THD) of space vector modulation is discussed. This method is also extended to a new stator flux based space vector modulation whereby the error flux vector is used to directly obtain the inverter switching states. Programs illustrating the method were written using MATLAB to calculate and display the THD of voltage controlled space vector modulation and stator flux based space vector modulation. In addition, the performances of modern PWM methods are analyzed and the simulation results are presented. The total harmonic distortion of space vector modulation for operation in the normal range to the overmodulation range is also presented in the paper.

I. INTRODUCTION

Voltage source inverters employ pulse-width modulation (PWM) switching to obtain variable voltage at variable frequency from DC supply. PWM is implemented using sine-triangle intersection technique or the space vector modulation technique. Recently, a stator flux based space vector modulation adopting direct flux control has been introduced in (Tripathi, 2002) for application in three phase AC machines. The method of PWM is inherent to the stator flux vector control and is designed for discrete time systems. In this modulation scheme, the error in torque is used to determine the position of the reference stator flux vector, and the error vector between this reference and the actual stator flux vector will determine the inverter switching state vectors.

This project aims to analyze the performance of space vector modulation and to carry out a simulation of its performance using MATLAB. Total harmonic distortion is used as an indication of the performance of the PWM technique. The method of calculating THD is obtained and modified from (Narayanan, 1999). A program illustrating this method is then written for voltage controlled space vector modulation. This program calculates the mean square value of the harmonic flux ripples only for a sector (first 60°) because of the symmetry of the space vector diagram as shown in figure 1. In addition, the calculation is done for a subcycle due to the symmetrical switching sequence of space vector modulation techniques.

Similarly, another program is written to calculate and display the THD of the stator flux based space vector modulation. This program is constructed based on a MATLAB program (Tripathi, 2002) that demonstrates stator flux based space vector modulation. In the following sections, the method for calculating THD is presented, and the performance of space vector modulation techniques under voltage control and direct flux control are illustrated and evaluated.

\textsuperscript{1} Student
\textsuperscript{2} Research scholar
\textsuperscript{3} Supervisor
II. PERFORMANCE OF VOLTAGE CONTROLLED SPACE VECTOR MODULATION

When the reference voltage vector is in the first sector of the hexagon, the voltage ripple vectors and harmonic flux ripples over a subcycle, τₜ, are shown in figure 2. The voltage ripple vectors and harmonic ripples are represented in a revolving d-q reference frame.

![Space Vector Diagram](image)

![Harmonic Flux Ripple Diagram](image)

Figure 1: The space vector diagram

Figure 2: Harmonic flux ripple in the first sector of the hexagon

The mean square value of the harmonic flux ripple is used to determine the performance of space vector modulation. Normalized voltages and times are used in the calculations. The modulation index is the normalized fundamental voltage and is defined as

$$m = \frac{u_1}{u_{six-step}} \quad \text{where} \quad u_{six-step} = \frac{2u_{dc}}{\pi}$$

The switching times for the active states and the zero states from normal range up to six-step operation can be obtained from (Holtz, 1991). The values of the d-axis and q-axis components of the flux ripples in the normal range are as follows.

$$\psi_{q1} = -m\tau_0$$

$$\psi_{q2} = -m\tau_0 + \left(\frac{\pi}{3}\cos\alpha - m\right)\tau_1$$

$$\psi_{q3} = m\tau_1 = m\tau_0 = -\psi_{q1}$$
The figure below shows the THD of the various space vector modulation techniques. In the normal range, it can be seen that SVPWM (Space Vector Pulsewidth Modulation) has superior performance at low modulation index whereas DPWM methods has better performance at higher modulation index.

\[ \psi_{d2} = \frac{\pi}{3} \tau_1 \sin \alpha \]  
\[ \psi_{q, rms}^2 = \frac{1}{\tau_s} \int_0^{\tau_s} \psi_q^2 d\tau = \frac{1}{3} \psi_{q1}^2 + \frac{1}{3\tau_s} \psi_{q2}^2 (\tau_1 + \tau_2) + \frac{1}{3\tau_s} \psi_{q1} \psi_{q2} (\tau_1 - \tau_2) \]  
\[ \psi_{d, rms}^2 = \frac{1}{\tau_s} \int_0^{\tau_s} \psi_d^2 d\tau = \frac{1}{3\tau_s} \psi_{d2}^2 (\tau_1 + \tau_2) \]  
\[ \psi_{rms}^2 = \psi_{q, rms}^2 + \psi_{d, rms}^2 \]  

The figure below shows the THD of the various space vector modulation techniques. In the normal range, it can be seen that SVPWM (Space Vector Pulsewidth Modulation) has superior performance at low modulation index whereas DPWM methods has better performance at higher modulation index.

In overmodulation mode I, when the reference voltage is within the boundary of the hexagon, a higher fundamental component is required to compensate for the reduced fundamental component during the instances when the reference voltage trajectory is outside the hexagon. Hence the normalized times are modified as shown in (Tripathi, 2002) and the harmonic ripples in overmodulation mode I are displayed as follows.

\[ \psi_{q, rms}^2 = \frac{1}{\tau_s} \int_0^{\tau_s} \psi_q^2 d\tau = \frac{1}{3} \psi_{q1}^2 + \frac{1}{3\tau_s} \psi_{q2}^2 (\tau_1 + \tau_2) + \frac{1}{3\tau_s} \psi_{q1} \psi_{q2} (\tau_1 - \tau_2) \]  
\[ \psi_{d, rms}^2 = \frac{1}{\tau_s} \int_0^{\tau_s} \psi_d^2 d\tau = \frac{1}{3\tau_s} \psi_{d2}^2 (\tau_1 + \tau_2) \]  
\[ \psi_{rms}^2 = \psi_{q, rms}^2 + \psi_{d, rms}^2 \]  

Figure 3: Total harmonic distortion of modern PWM methods

Figure 4: Harmonic flux ripple in overmodulation mode I
In overmodulation mode II, the number of switching per subcycle depends on the holding angle, $\theta_h$. The modified reference voltage vector remains at the vertices when the reference angle is less than $\theta_h$ or greater than $(\pi/3 - \theta_h)$ but less than $\pi/3$. At other angles in the first sector, there will be two switching per subcycle. The harmonic distortion is calculated as follows.

\[ \psi_{q1} = \left(\frac{\pi}{3} \cos \alpha - m_i\right) \tau_1 \]  \hspace{1cm} (9) \]

\[ \psi_{d2} = \frac{\pi}{3} \tau_1 \sin \alpha \]  \hspace{1cm} (10) \]

\[ \psi_{rms}^2 = \frac{1}{\tau_s} \int_0^{\tau_s} \psi_{q1}^2 d\tau + \frac{1}{\tau_s} \int_0^{\tau_s} \psi_{d2}^2 d\tau = \frac{1}{3\tau_s} \psi_{q1}^2 (\tau_1 + \tau_2) + \frac{1}{3\tau_s} \psi_{d2}^2 (\tau_1 + \tau_2) \]  \hspace{1cm} (11) \]

In overmodulation mode II, the number of switching per subcycle depends on the holding angle, $\theta_h$. The modified reference voltage vector remains at the vertices when the reference angle is less than $\theta_h$ or greater than $(\pi/3 - \theta_h)$ but less than $\pi/3$. At other angles in the first sector, there will be two switching per subcycle. The harmonic distortion is calculated as follows.

\[ 0 \leq \theta \leq \theta_h \]  \hspace{1cm} (12) \]

\[ \psi_{q1} = \left(\frac{\pi}{3} \cos \alpha - m_i\right) \tau_1 \] \hspace{1cm} (13) \]

\[ \psi_{d2} = \frac{\pi}{3} \tau_1 \sin \alpha \] \hspace{1cm} (14) \]

\[ \psi_{rms}^2 = \frac{1}{3\tau_s} \psi_{q1}^2 \tau_1 + \frac{1}{3\tau_s} \psi_{d2}^2 \tau_1 \] \hspace{1cm} (15) \]

\[ \theta_h \leq \theta \leq \frac{\pi}{3} - \theta_h \] \hspace{1cm} (16) \]

\[ \psi_{q1} = \left(\frac{\pi}{3} \cos \alpha - m_i\right) \tau_1 \] \hspace{1cm} (17) \]

\[ \psi_{d2} = \frac{\pi}{3} \tau_1 \sin \alpha \] \hspace{1cm} (18) \]

\[ \psi_{rms}^2 = \frac{1}{3\tau_s} \psi_{q1}^2 (\tau_1 + \tau_2) + \frac{1}{3\tau_s} \psi_{d2}^2 (\tau_1 + \tau_2) \] \hspace{1cm} (19) \]

\[ \frac{\pi}{3} - \theta_h \leq \theta \leq \frac{\pi}{3} \] \hspace{1cm} (20) \]

Normalizing against the harmonic distortion at six-step, the THD of SVPWM from normal range up to six-step is shown below. We can see that the harmonic distortion decreases slightly when the reference voltage enters into the overmodulation range. This is due to the lesser switching required per subcycle while the effective switching frequency is maintained. The distortion then increases rapidly, reaching a unity value at six-step operation.
III. PERFORMANCE OF STATOR FLUX BASED SPACE VECTOR MODULATION

The same approach as above is used to calculate the THD under this control technique. The normalized switching times of the active and zero states can be obtained from (Tripathi, 2002). The figures below show the THD of SVPWM for different subcycle time intervals. We can see that the curve is smoother and the distortion in overmodulation range is lower at a smaller Ts value. Thus, we can conclude that the THD is lower at high switching frequencies.

Comparing figure 5 and 6, we can observe that the THD of SVPWM under voltage control and direct flux control have approximately the same waveforms. Hence both control methods produce similar performance in terms of harmonic distortion.

IV. CONCLUSION

From the simulations, we can conclude that SVPWM provides a superior performance at low modulation index whereas the DPWM methods have a better performance at high modulation index. In addition, the distortion tends to decrease when entering the overmodulation range before increasing rapidly to unity at six-step operation. Space vector modulation under voltage control and direct flux control provide the same performance in terms of harmonic distortion. We can also observe that the harmonic distortion is lower at higher frequencies.
V. REFERENCES


Tripathi, Anshuman (2002), MATLAB program illustrating stator flux based space vector modulation.