Keywords: Printed waveguide; planar helix; printed slow-wave structure; surface waves; circularly polarised fields.

Abstract

In this paper, a new type of waveguide structure has been investigated. The structure can be fabricated using printed-circuit techniques. The characteristic equations for the different modes of the structure have been derived and the dispersion characteristics have been obtained using MATLAB. The slow-wave nature of the structure, the cut-off behaviour of different modes, and the effects of variation in the physical parameters have been examined.

1 Introduction

There is a trend towards use of high microwave frequencies for wireless communications, radar, and automotive applications. At such high frequencies, one can achieve high data rates and small component-size. However, at such frequencies, conventional waveguides are expensive to fabricate, and printed transmission lines such as microstrip and coplanar waveguide suffer from radiation and surface-wave phenomena. The main objective of this work is to investigate a new type of structure, which can be fabricated using printed-circuit techniques and is free from the problem of leakage through surface-wave phenomena.

This work focuses on a planar structure employing a pair of anisotropic screens; each screen is perfectly conducting in one direction and perfectly insulating in the perpendicular direction. Such a screen is called a unidirectionally conducting (UC) screen. As shown in Fig. 1, the structure studied here comprises a pair of parallel UC screens. The regions inside and outside the screens consist of different dielectric materials in general.

1 Figure 1: Structure of “planar helix”

Figure 2: Simplified model of the structure

Figure 2: Simplified model of the structure

The structure with its constructional parameters is shown in Figure 2. The distance between the pair of screens is \( b \). The media inside and outside the screens have relative permittivity of \( \varepsilon_r2 \) and \( \varepsilon_r1 \), respectively. The screens are confined in the transverse direction. This confinement can be in the form of a metal waveguide, parallel conducting planes or electronic (photonic) bandgap. For the case studied here, the separation between the confining conducting planes is \( a \). As shown in Fig. 3, the directions of conduction of the two screens make equal and opposite angles \( \alpha \) with the direction of wave propagation. In Fig. 3, \( z \) is the direction of propagation; \( y' \) and \( y'' \) are the directions of conduction for the top and bottom screens.

1 Figure 3: Directions of conduction
A pair of UC screens is found to have electromagnetic guiding properties that have considerable similarities with those of a circular helix. Therefore, this structure is also referred to as a planar helix [1].

Some properties of a pair of UC screens without transverse confinement (the structure is infinite in the transverse direction, i.e., y direction in Figure 3) have been examined in detail. The dispersion characteristics have been worked out when the media inside and outside the screens are different. In [1], it was found that, the structure without confinement has some propagating modes that are very similar to those of a circular helix, so these modes are called “helix modes”. Other modes supported by the structure are essentially similar to those of a dielectric slab guide. It was also found that the “helix modes” are strongly affected by the directions of conduction of the screens (angle $\alpha$) while the other modes are almost unaffected. Further, the fields supported by this structure were found to be circularly polarised. The applications of this structure in ferrite phase shifters [2] and travelling-wave tubes [3] have been studied in some detail.

Properties of non-radiating dielectric slab (NRD) (dielectric slab with confinement) have also been investigated in detail [4]. In an NRD, a dielectric slab is confined transversely between a pair of conducting planes. The spacing between the conducting planes is kept small enough to keep the dominant mode below cut-off outside the slab. Further, the operating mode is so selected as to have a low conductor loss. The NRD has been shown to have applications in a number of waveguide components as well as leaky-wave antennas [5].

The structure studied here is expected to have some properties similar to those of the infinite planar helix, and some properties similar to those of NRD. In addition, a strong advantage of the planar helix is its potential for fabrication on a dielectric substrate with printed-circuit techniques. So, it has the potential of evolving into a new kind of low-cost waveguiding/radiating structure for high microwave frequency applications.

Section 2, “Characteristic equations”, describes briefly the method of analysis and gives the characteristic equations for symmetric and anti-symmetric solutions. Section 3, “Method of solution using MATLAB”, explains the method adopted to solve the characteristic equations. The basic idea and the flow chart of the program are given in this part. Section 4, “Dispersion characteristics”, shows some of the important results obtained in this work. The dispersion characteristics ($k_0$ vs. $\beta$ diagram) are shown. The results are compared with the dispersion characteristic of non-radiating dielectric slab guide (NRD). The cut-offs of different modes are discussed. The effects of variations in angle of conduction, $\alpha$, and the dimensions of the structure (ratio b/a) are also examined. Section 5 mentions the conclusions of the project.
components can be represented in terms of scalar potentials $\psi^E$ and $\psi^H$ as following:

$$E_{zi} = \frac{j k_i^2 - \beta^2}{\beta} \psi_i^E (x, y) e^{-j k_i z} \quad (1)$$

$$H_{zi} = \frac{j k_i^2 - \beta^2}{\beta} \psi_i^H (x, y) e^{-j k_i z} \quad (2)$$

$$E_{yi} = \left( \frac{\omega \mu_0}{\beta} \frac{\partial \psi_i^H}{\partial x} + \frac{\partial \psi_i^E}{\partial y} \right) e^{-j k_i y} \quad (3)$$

$$H_{yi} = \frac{\omega \varepsilon_i}{\beta} \frac{\partial \psi_i^E}{\partial x} + \frac{\partial \psi_i^H}{\partial y} \quad e^{-j k_i y} \quad (4)$$

$$E_{zi} = \left( \frac{\partial \psi_i^E}{\partial x} + \frac{\sigma \mu_0}{\beta} \frac{\partial \psi_i^H}{\partial y} \right) e^{-j k_i y} \quad (5)$$

$$H_{zi} = \left( \frac{\omega \varepsilon_i}{\beta} \frac{\partial \psi_i^E}{\partial x} + \frac{\partial \psi_i^H}{\partial y} \right) e^{-j k_i y} \quad (6)$$

where $k_i^2 = \sigma \varepsilon_0 \mu_0 k_i^0, \ v_n=1,2,3…$

Medium 2:

$$\psi_2^E = A_2 \sin(k_y y) \cosh(k_{x2} x), x \leq b/2$$

$$\psi_2^H = B_2 \cos(k_y y) \sinh(k_{x2} x)$$

Medium 1:

$$\psi_1^E = A_1 \sin(k_y y) e^{-k_{x1}(x-b/2)}, x \geq b/2$$

$$\psi_1^H = B_1 \cos(k_y y) e^{-k_{x1}(x-b/2)}, x \geq b/2$$

where $k_y = \frac{2n \pi}{a}, n=1,2,3…$

$$k_{x1}^2 = k_y^2 + \beta^2 - k_1^2$$

$$k_{x2}^2 = k_y^2 + \beta^2 - k_2^2$$

Substitution of the above potentials in equations (1)–(6) yields all the field components. The field components are further subjected to the following boundary conditions:

(i) $E_{yi} \left( \frac{b}{2}, y \right) = E_{zy} \left( \frac{b}{2}, y \right)$

(ii) $E_{zi} \left( \frac{b}{2}, y \right) = E_{zz} \left( \frac{b}{2}, y \right)$

(iii) $H_{yi} \left( \frac{b}{2}, y \right) = H_{zy} \left( \frac{b}{2}, y \right)$

or $H_{yi} \cos \alpha + H_{zi} \sin \alpha = H_{zy} \cos \alpha + H_{zz} \sin \alpha$

at $x=b/2$

(iv) $E_{yi} \left( \frac{b}{2}, y \right) = E_{yz} \left( \frac{b}{2}, y \right) = 0$

or $E_{yi} \cos \alpha + E_{zi} \sin \alpha = 0$ at $x=b/2$

This leads to the following characteristic equation:

Symmetric solutions:

$E_i$ is even with respect to $x$ and $y$:

In this case, we look for solutions that satisfy:

$$\psi^E (x, y) = -\psi^E (-x, -y)$$

$$\psi^H (x, y) = -\psi^H (-x, -y)$$

$$\psi^E (x, \pm \frac{a}{2}) = 0$$

$$\frac{\partial \psi^H (x, y)}{\partial y} \bigg|_{y=\pm a/2} = 0$$

To satisfy the above conditions, the scalar potentials are chosen as following:
\[
\frac{\tanh(k_{x_2} b)}{2} \left\{ \frac{k_1^2 - \beta^2}{k_2^2 - \beta^2} + \frac{\tan^2(\alpha) (k_1^2 - \beta^2)(k_1^2 - \beta^2)}{\beta^2 k_y^2} \right\} = \\
\frac{k_1^2 k_{x_1} - 1}{k_{x_1}} \left\{ 1 + \tan^2(\alpha) \frac{(k_1^2 - \beta^2)^2}{\beta^2 k_y^2} \right\} + \\
\frac{k_2^2 k_{x_2} k_1^2 - \beta^2}{\beta^2 k_y^2} \frac{k_1^2 - \beta^2}{k_2^2 - \beta^2} \coth(k_{x_2} b / 2)
\]

\[\psi_x^E = A_1 \cos(k_y x) \sinh(k_{x_2} x), \quad x \leq b/2\]

\[\psi_x^H = B_1 \sin(k_y x) \cosh(k_{x_2} x)\]

Medium 1:

\[\psi_1^E = A_1 \cos(k_y x) e^{-k_{x_2} (x-b/2)}, \quad x \geq b/2\]

\[\psi_1^H = B_1 \sin(k_y x) e^{-k_{x_2} (x-b/2)}\]

where \( k_y = \frac{(2n-1)\pi}{a} \), \( n = 1, 2, 3 \ldots \)

\[k_{x_1}^2 = k_y^2 + \beta^2 - k_1^2\]

\[k_{x_2}^2 = k_y^2 + \beta^2 - k_2^2\]

Then, from equations (1)-(6), all the field components can be found. Subject to the boundary conditions mentioned earlier, the characteristic equation is found to be:

**Anti-Symmetric solutions:**

In this case, we have two solutions:

\( E_z \) is even with respect to \( x \) and odd with respect to \( y \)

\( E_z \) is odd with respect to \( x \) and even with respect to \( y \)

The procedure to find the characteristic equations is very similar to the method adopted to find the characteristic equations for the symmetric case. These solutions can be obtained from symmetric equations with a minor modification.

### 3 Method of solution using MATLAB

The main MATLAB function employed in this project is `fsolve` of the Optimisation Toolbox.

In the characteristic equations, all the variables can be normalized by multiplying them with a factor of \( b/2 \). Then, for each value of normalized frequency \( (k_0 b)/2 \), we find out the possible solutions for normalized decay constant for medium 2 \( (k_2 b)/2 \); this leads to the different values of the normalized phase constant \( (b\beta)/2 \). Therefore, different modes can be found. To find out different solutions for each value of \( k_0 b/2 \), the `fsolve` function is used with different initial guesses. The graphic capability in MATLAB is helpful in finding out different initial guesses for `fsolve` function.

During the process of root finding, sometimes, we need to find the complex roots of the characteristic equation. Since `fsolve` function does not have the capability to return complex root, we have to separate real and imaginary parts of the equation. By solving these two equations simultaneously, complex roots can be determined.

Let \( F(\gamma) = 0 \) be the equation we have to solve. We need to find the complex solution in the form: \( \gamma = \alpha + j\beta \). Then, we have:

\[ F(\gamma) = F(\alpha + j\beta) = F_1(\alpha, \beta) + jF_2(\alpha, \beta) \]

where

\[ F_1(\alpha, \beta) = \text{Re}[F(\alpha + j\beta)] \]

\[ jF_2(\alpha, \beta) = \text{Im}[F(\alpha + j\beta)] \]

From \( F(\alpha + j\beta) = 0 \) we have:

\[
\begin{align*}
F_1(\alpha, \beta) &= 0 \\
F_2(\alpha, \beta) &= 0
\end{align*}
\]

By solving \( F_1 \) and \( F_2 \) simultaneously using `fsolve` function, the complex root \( \gamma = \alpha + j\beta \) can be found.

### 4 Dispersion characteristics

4.1 For \( \alpha = 30^\circ \), \( b/a = 0.5 \), \( \varepsilon_1 = 1 \), \( \varepsilon_2 = 2.55 \), the dispersion characteristics of the confined planar helix, for the symmetric solutions, are shown in Figure 5.
Figure 5: Dispersion characteristic of confined planar helix with $\alpha = 30^\circ$, $b/a = 0.5$, $\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 2.55$

As can be seen from Figure 5, a number of modes can propagate in the structure. There are essentially two types of modes: helix modes and waveguide modes. As can be seen, all the waveguide modes are constrained below the straight line $\beta = k_0 \sqrt{\varepsilon_{r2}}$ while the helix modes are not. Helix modes are strongly affected by the angle of conduction of the UC screens as shown later.

4.2 Comparison with NRD (non-radiating dielectric slab guide) dispersion characteristic

Figure 6 shows the dispersion characteristics of NRD guide with $b/a = 0.5$, $\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 2.55$ (which are the same as for the confined planar helix shown in Figure 5). As can be seen from Figures 5 and 6, the waveguide modes of the confined planar helix are almost the same as the propagation modes of NRD. The only difference is that, in confined planar helix, there do exist some special modes, which are called “helix modes” which the NRD does not have.

4.3 Cut-off frequencies of different modes:

As can be seen from dispersion characteristics of the confined planar helix (Figure 5), each mode has a certain cut-off frequency. These cut-off frequencies may not be easily obtained analytically since the characteristic equation is complicated.

Figures 7a and 7b show $k_{x1}$ vs $k_0$ and $k_{x2}$ vs $k_0$ for the confined planar helix with $\alpha = 30^\circ$, $b/a = 0.5$, $\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 2.55$, respectively. From these plots, we can predict the cut-off conditions for different modes.

(i) For the helix modes, the cut-off occurs when $k_{x1}$ and $k_{x2}$ approach zero. Both $k_{x1}$ and $k_{x2}$ are real for these modes.

(ii) For waveguide modes, cut-off occurs when $k_{x1}$ becomes 0. In this case, $k_{x2}$ is imaginary and remains almost constant over the whole range of frequency.

4.4 Effects of variations in $\alpha$, $b/a$

Figure 8 shows the dispersion characteristics of the confined planar helix with all the parameters the same as the previous case, except that the angle of conduction $\alpha$ is changed to $10^\circ$. As can be seen, all the helix modes change tremendously while the waveguide modes remain almost the same. Therefore, we can conclude that the angle of conduction of the UC screens, $\alpha$, has a strong effect on the helix modes of
propagation, but has very little effect on the waveguide modes. So, in this respect, the structure studied here (a confined pair of UC screens) is similar to a helix. That is the reason why it is called confined planar helix.

as that of $\alpha$. In these two cases, the ratio $b/a$ is relatively small. So, the waveguide modes are almost the same for different values of $n$. And for the helix modes, EVEN and ODD solutions are almost identical (especially at high frequency).

5 Conclusion

In this paper, a new kind of waveguide structure has been investigated. Its dispersion characteristics are examined to find out the possible modes of propagation and their cut-off behaviour. A considerable similarity with a helix and NRD has been brought out clearly. These properties, together with its strong advantage in fabrication are expected to make the structure popular. The structure has the potential to evolve into a new kind of printed wave-guide structure for high microwave frequencies. Further investigations will be carried out in future with respect to the nature of the fields, loss characteristics, frequency range for single-mode propagation, excitation etc.

References


