FILTER DESIGN FOR PULSE WIDTH MODULATED INVERTERS

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Abstract
With the use of Simulink, I am able to perform the modeling and analysis of the voltage source inverter (VSI), LC filter, load as specified and the pulse width modulator, which will be discussed in the body of this report. In addition, the overall system is simulated using a Proportional-Integral-Derivative controller with variable set points to adjust the output voltage to the load. Cost optimization is also discussed to inform the readers of the current costs of the inductors, capacitors and resistors available today.

1 Introduction
Designers have to embark on filter design research because they need to know the limitations it has for the application at hand. Basically, a filter, in this case has to have as low loss in power as possible and meet the total harmonic distortion of five per cent maximum under 10% to 100% of full load voltage applied to the load. Through mathematical calculations, designers are able to come up with a low pass filter that far meets the specifications given. However, there are other factors to consider like the use of readily available components to form the filter, cost optimization and size minimization. When such factors are considered, designers are left with compromises, which will influence their design and downplay the performance of the filter.

The first factor - meeting the specifications is on top of the priority list that affects the filter design. Just consider the two main parameters that are of vital use to designers – maximum total harmonic distortion as well as voltage output to the load. Now, the formula for total harmonic distortion is as follows,

\[ \text{THD in (\%)} = \sqrt{\sum (V_n^2 + V_{n+1}^2 + V_{n+2}^2 + \ldots )/ V_i} \times 100\% \]

where \( V_n \) is the nth harmonic (exclude fundamental) voltage in RMS. \( V_i \) is the fundamental harmonic voltage in RMS.

From the formula, we know that if the magnitude of fundamental harmonic voltage is changed, THD will increase for a filter used in the inverter application. The remaining harmonics will be roughly the same regardless of the magnitude of fundamental harmonic voltage. This is vital when output voltage is variable from 10% to 100% of full-scale voltage.

Although the power loss of the filter is not mentioned in the specifications, designers have to assume that the filter consumes power that is negligible relative to the loads’ due to the internal resistance of the inductor that forms the LC filter. Power loss of a filter is related to its Q or quality factor and low power loss would indicate a high Q filter. A Q factor of 20 is used in the filter design and will be discussed later.

After satisfying the given specifications, the next thing to do is to think about its component values. Note that this is closely related to cost optimization as choosing readily available values do help reduce the total cost of the filter. The filter which consists of inductors and capacitors and to a certain extent, resistors of very small value to provide damping of the
system whose values are derived from mathematical calculations must match the ones available as close as possible. Otherwise, the achieved results would stray from the initial calculated ones. Furthermore, should one component be relatively cheaper to obtain than the other, mathematical manipulation should be used such that much of the values are shifted to that component. In this case, the capacitors are a lot cheaper than the inductor; so it is advisable to increase the capacitor values oppositely decrease the inductor values to the limits without compromising the specifications.

Size minimization is inevitably achieved when cost has been optimized. For an LC filter, having smaller inductor values would imply smaller number of turns and thus, smaller in size than those having high values of equal current ratings. Surprisingly, size and cost do go hand in hand in this filter design and even though values of capacitor change, it is relatively smaller in size than the inductor even in its highest available value.

Such are the factors considered in designing the filter, which prove the importance of filter research. Without it, mathematical calculations would prove inefficient in practical applications although they may be relied on solely. The inverter is used in many power applications such as AC motors, DC motors and uninterruptible power supplies. It is an electronic device, which utilizes the switching of an array of power transistors such as IGBTs, MOSFETs and BJTs, to convert a fixed DC voltage to a three-phase AC voltage with variable magnitude and frequency. Due to the use of PWM (pulse width modulation) technique, the inverter usually generates voltage harmonics in a chopped form instead of purely sinusoidal. These harmonics will have effects on the load such as additional power loss and noise interference.

The main objective of this project is to design a three-phase filter, which can reduce or minimize the harmonics generated by the inverter given the specifications as follows:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Frequency</td>
<td>50Hz</td>
</tr>
<tr>
<td>Max. Output Voltage</td>
<td>240V</td>
</tr>
<tr>
<td>Max. Output Power</td>
<td>15KVA</td>
</tr>
<tr>
<td>Switching Frequency</td>
<td>15kHz</td>
</tr>
<tr>
<td>Modulation Technique</td>
<td>Sinusoidal-Triangle Modulation</td>
</tr>
<tr>
<td>DC Input Voltage</td>
<td>Determined by the designer</td>
</tr>
<tr>
<td>Load Requirements</td>
<td></td>
</tr>
<tr>
<td>Rated Voltage</td>
<td>220V ± 2%</td>
</tr>
<tr>
<td>(Voltage Feedback Control)</td>
<td></td>
</tr>
<tr>
<td>Max. THD</td>
<td>5% (Use Fourier Series)</td>
</tr>
<tr>
<td>Power Consumption</td>
<td>1KW – 10KW</td>
</tr>
<tr>
<td>Power Factor</td>
<td>0.9 lagging (fixed)</td>
</tr>
</tbody>
</table>

Filter Design:

Low-Pass Filter with size optimization

Through proper design of the filter, the costs as well as size of the inductors and capacitors are optimized. This is possible by reducing the inductor value and increasing capacitor value. However, components availability limits the capacitor value to about tens of microfarads, typically about 80μF. In my case, the reactance of the filter is reduced to 0.03 p.u., indicating how small the value and therefore size of the inductor is.

There is, however a new breed of capacitors called ‘ultra capacitors’ which are for continuous, AC
applications such as in inverter systems. These capacitors have values of several hundreds of microfarads and because new implies high costs for purchasing them, they are not used in our design. Perhaps ultra capacitors can be used later when its cost are more affordable for wide usage.

2. Methodology

2.1 The Overall Circuit Diagram

2.1.1 Circuit Theory

The inverter concerned is a three phase DC to AC voltage converter. It supplies power to a three phase star connected resistive-inductive (RL) load, keeping the required voltage output constant. The voltage waveform from the inverter contains odd harmonic contents except those multiples of three and it will not be able to satisfy the required total harmonic distortion required by the load to operate properly. Therefore, a 3-phase filter is inserted in between such that the power delivered to the load remains approximately the same without much loss of power. Here, an LC low pass filter in star configuration is used.

2.1.2 Overall System Model

Figure 1: A Three-Phase Inverter System Block Model.

Figure 1 shows the overall system incorporating a voltage feedback control PID controller. The output voltage to the load can be varied from 22V to 220V rated voltage with error of ±2%. A constant of 8.25 is a maximum value set point that corresponds to full rated voltage of 220V. A slider gain in per unit (p.u.) can be adjusted such that, for example, 0.5 p.u. corresponds to 50% of full rated voltage, numerically 110V and 0.1 p.u. corresponds to 10%, which is 22V. The system basically contains four parts and they are:

1) Voltage Source Inverter (VSI).
2) Sinusoidal Pulse Width Modulator (SPWM).
3) Three Phase Star Connected Balanced Resistive-Inductive (RL) Load.
4) Three Phase Star Connected Low Pass Filter.
5) Proportional-Integral-Derivative (PID) Feedback Controller.

2.2 Voltage Source Inverter (VSI)

2.2.1 Theory

Figure 3: Three-Phase Voltage Source Inverter.

Shown in Figure 3, the power circuit of a three-phase voltage source inverter is obtained by adding a third leg to the single-phase inverter. Assuming that of the two power switches in each leg (phase) of the inverter, one and only one is always on, that is, neglecting the time intervals when both switches are off (blanking time), three switching variables, a, b and c can be assigned to the inverter. A state of the inverter is assigned as abc, making for a total of eight states, from state 0, when all output terminals are clamped to the negative DC bus, through state 7 when they are clamped to the positive bus.

It is easy to show that the instantaneous line to line output voltages; $V_{AB}$, $V_{BC}$ and $V_{CA}$ are given by

\[
\begin{bmatrix}
V_{AB} \\
V_{BC} \\
V_{CA}
\end{bmatrix} = \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]  
(1)
In a balanced three-phase system, the instantaneous line to neutral output voltages, $V_{AN}$, $V_{BN}$ and $V_{CN}$ can be expressed as

\[
\begin{bmatrix}
V_{AN} \\
V_{BN} \\
V_{CN}
\end{bmatrix} = \frac{1}{3}
\begin{bmatrix}
1 & 0 & -1 \\
-1 & 1 & 0 \\
0 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
V_{AB} \\
V_{BC} \\
V_{CA}
\end{bmatrix}
\]  \hspace{1cm} (2)

which, when combined with Eq. (1), yields

\[
\begin{bmatrix}
V_{AN} \\
V_{BN} \\
V_{CN}
\end{bmatrix} = \frac{V}{3}
\begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]  \hspace{1cm} (3)

Eqs. (1) and (3) allow easy determination of the line to line and line to neutral output voltages for all states of an inverter.

2.2.2 Inverter Model

In Simulink, the inverter is modeled with reference to Eq. (3) in order that the line to neutral output voltages be obtained as illustrated in Figure 5.

![Figure 5: Model of the Three-Phase Voltage Source Inverter.](image)

The output voltage waveforms of the inverter are chopped as shown in Figure 6(a) when the waveforms of switching variables $a$, $b$ and $c$ through the PWM are input to the inverter. The waveforms $a$, $b$ and $c$ in Figure 6(a) are produced by the PWM when the feedback loop is set to maintain at 10% of full load rated voltage at 22V. The difference in voltage waveforms between Figure 6(a) and 6(b) is observed. Figure 6(b) illustrates the output voltage waveforms when the feedback loop is set to maintain at full load rated voltage. The voltages settings to the load can only be changed through the set point found in the feedback controller and are not affected by changes in the load.

2.3 Pulse Width Modulator (PWM)

2.3.1 Theory

As stated earlier, the PWM uses a carrier comparison scheme to produce the gating signals to the inverter. Since in each leg, only one IGBTs are on at any one time, we only consider one gating signal per leg and call it the switching variable. Since there are six IGBTs, there should be 3 switching variables - $a$, $b$ and $c$.

2.3.2 PWM Model

The model used in the system is shown in Figure 7.

![Figure 7: Three-phase sinusoidal pulse width modulator utilizing the carrier comparison scheme.](image)

Note that each of the sub-blocks depicting sine waves are 120° out of phase from each other. The sine waves used are of constant magnitude and in order to adjust their magnitudes simultaneously so that the PWM output changes according to the “triangulation” scheme, multipliers are added to each comparators. These multipliers performs product between a constant voltage to the corresponding sine waves. In turn, the feedback controller produces the constant voltage. Therefore, an increase in the sine waves’ magnitudes would result in increase of voltage supplied to the load per phase.

Specifically, the sine waves’ magnitudes are compared with the magnitudes are compared with the magnitude of the triangular wave which is held fixed at all times to provide some form of measure termed as the modulation index. Modulation index is the ratio of the modulating signals’ magnitude to the carrier’s magnitude. In
In this case, the carrier is the triangular wave and the modulating signals are the variable sine waves.

Now, the carrier’s magnitude is set to 10V peak to peak, limiting the sine waves’ magnitude to 10V peak to peak. This means that the output voltage to the load can be varied from 0V to 220V when the PWM’s sine wave sub-blocks are varied from 0V to 10V respectively. Since the magnitude of the sine waves can be adjusted from a feedback controller, the load voltage can be varied at any set point possible.

In addition, the switching frequency of the PWM is the carrier frequency signal, which is 15kHz. The modulating frequency is 50Hz. So, due to the presence of these frequencies, the voltage output from the inverter would have dominant harmonics at these frequencies.

2.4 Three Phase Star Connected Balanced RL Load

2.4.1 Theory and Load Model

The load is simply modeled as combinations of three single-phase transfer function as shown in Figure 8.

![Figure 8: Model of Three Phase Star Connected Balanced RL Load.](image)

Each sub-block represents per phase impedance and the output, through ohm’s law, is the corresponding single phase current. Through the specifications, we know that the load’s power consumption ranges from 1KW to 10KW with power factor held fixed at 0.9 lagging. It is easy to prove that under these conditions and with given rated voltage that the impedance per phase of the load ranges from $39.2 + j19 \ \Omega$ to $3.92 + j1.9 \ \Omega$ corresponding to the range of power consumption from 1KW to 10KW. In short, the impedance changes proportionally with power consumption.

After the impedance per phase has been determined, it is easy to obtain the values of L and R that forms the impedance transfer function of the Load Simulink block by dividing the imaginary part of impedance by $2\pi \times 50\text{Hz}$ to obtain L in henries and taking the real part as the value of R in ohms.

In order to simulate a changing L and R value corresponding to change in power consumption holding the power factor at 0.9 lagging, an isolated block has been created to perform these calculations for convenience named as Pcon. By just specifying the power consumption in this block, the R and L values are automatically displayed and users are able to set the load values of L and R manually, without hassle.

2.4.2 Results

The load current and voltage waveforms are already shown in Figure 2(b) – (e) for comparisons under various conditions.

2.5 Feedback Controller

2.5.1 Theory

The function of feedback control is to ensure that the voltage to the load is maintained at all times. Furthermore, if the set point is changed, it must be able to settle down the system to the new voltage setting. To do so, the feedback controller requires two elements to process the voltage signals for comparisons before taking any predictive action. The two elements are the envelope detector, usually a diode to rectify the three-phase voltage and a low pass (LP) filter to smoothen out the DC ripples. The output of the LP filter is a smooth DC voltage of a certain value, which is then compared to the set point DC voltage value to produce the measuring error. The deviation of this error determines from zero determines the response of the PID controller.
PID controller is used because it has very fast response time to a step input, stable and produces no or little overshoot if tuned properly which is best suited for this design project. The PID transfer function is given as \( G_c(s) = K_p(1 + \frac{1}{T_i s} + T_ds) \). In order to find the values of \( K_p, T_i \) and \( T_d \), Ziegler-Nichols step response method is used.

### 2.5.2 Controller Model

![Controller Model Diagram](image)

The controller model consists of a few sub-blocks as named in Figure 9. Detailed discussions on how it works will be discussed in the preceding section. Consequently, we are able to determine the delay time \( L_T \) that is found to be 0.0075s, the time constant of the system \( T \) that is known to be 0.02s and gain of the system \( K \) that is equal 8.25V.

Utilizing a Cohen-Coon turning rule, we are able to compute \( K_p, T_i \) and \( T_d \) using the following formulas:

\[
K_p = \frac{T}{KL_T\left(4/3 + L_T/4T\right)} \quad (4) \\
T_i = \frac{L_T\left(32T + 6L_T\right)}{13T + 8L_T} \quad (5) \\
T_d = \frac{4TL_T}{11T + 2L_T} \quad (6)
\]

After setting the three values in the PID, the plant is run at the same open loop normal operating conditions as earlier on and the output of the PID is captured on the scope. We need the voltage output from the controller to be 10V so that a steady state full rated voltage of 220V line to line is supplied to the load. To achieve this, a compensating gain of 1.709 at the output of the PID is added before the signal is feedback to the PWM. Thus, the loop is closed.

The whole closed looped inverter system is simulated over again under many set points from 10% to 100% of full load voltage and found to be stable. By trial and error, the transient response of the PID is further tuned to make the response faster with little overshoot.

Care is ensured that the steady state ripple is within tolerable limits specified such that the required supplied voltage to the load is within ±2%.

### 2.6 LC Filter

#### 2.6.1 Filter Theory

![Filter Theory Diagram](image)

As shown in Figure 12, the filter is a low pass type and is formed by combining a single phase LC low-pass filter into a three phase star configuration. The resistance \( R \) accounts for the internal resistance of the inductor and is very small, typically in micro-ohms.

Since in a three-phase filter, the ground is neutral, we can consider per phase of the filter in our calculations. Thus, we found that,

\[
V_{\text{in}}/V_{\text{out}} = \frac{1}{L/s^2 + RC + 1} \quad (7)
\]

for a single phase LP-LC filter.

The analysis of the filter is done in the frequency domain and for a typical low pass filter; the voltage gain is

\[
H(j\omega) = \frac{w_p^2}{s^2 + w_p^2/Q + w_p^2} \quad (8)
\]

where \( w_p \) is the natural oscillating frequency and \( Q \) is the quality factor. It can be proven that in the case of peaked resonances, or \( Q > 1/\sqrt{2} \), the frequency at which
\( H(jw)_{\text{max}} \) is maximized and the corresponding maximum are

\[
w/w_p = \sqrt{1 - 1/2Q^2} \tag{9}
\]

\( H(jw)_{\text{max}} = Q/\sqrt{1 - 1/4Q^2} \tag{10}\)

For sufficiently large \( Q \)'s, say \( Q > 5 \), we have \( w/w_p \cong 1 \) and \( H(jw)_{\text{max}} \cong Q \).

In addition, by rearranging Eq.(8) and comparing coefficients with (1),

\[
LC = 1/w_p^2 \tag{11}
\]

\[
RC = 1/Qw_p \tag{12}
\]

By solving simultaneous Eqs. (9) and (10), the \( L, C \) and \( R \) values of the filter are obtained.

2.6.2 Filter Model

Figure 13 shows the block function of the filter. Each transfer function is of the form as given in Eq.(8) but as a function of the s-domain, to represent a single phase LC filter and replicated for the second and third phase.

![Figure 13: Model of three phase low-pass LC filter.](image)

From the quotations of inductors and capacitors gathered and possible values of \( L \) and \( C \) readily available in the market, it is found that the capacitors cost cheaper than the inductor even in its highest value. So, from the product of \( LC \), it is logical that increasing the capacitor's value would decrease the inductor’s. However, the maximum typical and available capacitor value is 80uF. Thus, the corresponding inductor value is 2mH.

In per unit, the inductance of the filter using the maximum inductor value possible by the load (0.06044H) as base value, the filter per phase inductor in p.u. is about 3%, which is very good. It indicates the size of the inductor relative to the load in percentage.

The costs of the 80uF capacitor and 2mH, 30A rated inductor are about SGD$20 and SGD$100 respectively. Since the filter is three phase, the total cost of the filter is estimated to be SGD$360.

3 Conclusion

So far, we have analyzed how the whole inverter system operates and in detail how each blocks are designed in order that the specifications be achieved. The voltage and current waveforms of selected outputs and inputs are also shown to prove that the system works and is stable.

Finally, in the filter section, the cost of the filter is briefly discussed to inform the readers of the estimated cost of the optimized three-phase filter.

4 References:

