Complexity Reduction for MC-CDMA with MMSEC

Keli Zhang, Yong Liang Guan, and Qinghua Shi

***Abstract*—The minimum mean square error combining (MMSEC) scheme performs best among various equalization and combining schemes for downlink MC-CDMA. However, it is also most computationally complex to realize as it involves complex matrix inversion operation of a large matrix. In this paper, a new method is proposed to calculate the equalizer coefficient for MMSEC based on reduced-size matrices. It is shown that as long as the number of users is even, the matrix inversion size is reduced to at least N_c/2, where N_c is the number of subcarriers.**

***Index Terms*—MC-CDMA, MMSE.**

I. Introduction

Multicarrier code division multiple access (MC-CDMA) is a technique that combines direct sequence (DS) CDMA with orthogonal frequency division multiplexing (OFDM) modulation. It is one of the candidate technologies considered for the 4th generation wireless communication systems [1]. MC-CDMA transmits every data symbol on multiple narrowband subcarriers and utilizes cyclic prefix to absorb and remove inter-symbol interference (ISI) arising from frequency selective fading. As it is unlikely for all subcarriers to experience deep fade simultaneously, frequency diversity is achieved when the subcarriers are appropriately combined at the receiver. In [2] and [3], it is shown that MC-CDMA outperforms the conventional DS-CDMA and two other forms of CDMA with OFDM modulation, namely MC-DS-CDMA and multitone CDMA.

Several combining techniques have been proposed for MC-CDMA systems, namely orthogonality restoring combining (ORC), equal gain combining (EGC), maximal ratio combining (MRC) and minimum mean square error combining (MMSEC) [4]–[7]. Furthermore, there are two MMSEC variants, “MMSEC per carrier” and “MMSEC per user” [8]. The latter is the scheme that is considered in this paper as it performs best among all schemes mentioned above [4], [5], [8]. For simplicity of notation, it is referred as MMSEC hereafter. MMSEC performs best compared to all other schemes mentioned above, however, it is also most computationally complex to realize as it involves the matrix inversion operation of a large complex matrix. In this paper, we propose a way to reduce the matrix inversion dimension for calculating the MMSEC equalizer coefficient for downlink MC-CDMA. We have shown that Note that in the literature, many complexity reduction methods are proposed for MMSE channel estimator [9], [10] and DS-CDMA detector [11], [12]. Our proposed scheme, however, is specific to the MMSE combiner/equalizer for MC-CDMA downlink with Walsh-Hadamard spreading codes.

Keli Zhang is with Motorola Electronics Pte. Ltd., Techpoint #04-11/17, 10 Ang Mo Kio Street 65, Ang Mo Kio, Industrial Park 3, Singapore 569059 (e-mail: kelizhang@motorola.com).

Yong Liang Guan and Qinghua Shi are with the Positioning and Wireless Technology Centre, School of Electrical and Electronic Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798 (e-mails: {EYLGuan, qhshi}@ntu.edu.sg).

II. Signal Model and MMSEC equalizer coefficient

Considering a MC-CDMA system with N_u users, each of whom employs N_c subcarriers modulated with BPSK, the transmitted signal corresponding to the kth user can be expressed as

$$s_k(t) = \sum_{i=-\infty}^{\infty} \sqrt{\frac{2E_b}{N_c T_s}} \sum_{n=1}^{N_c} b_k(i) c_{k,n} u_{T_s}(t - iT_s) \cos(w_n t) \ , \tag{1}$$

where E_b and T_s are the bit energy and symbol duration respectively, $u_{T_s}(t)$ represents a rectangular waveform with amplitude 1 and pulse duration T_s, $b_k(i)$ is the ith transmitted data bit of user k, $c_{k,n}$ is the spreading code, $w_n = 2\pi f_0 + 2\pi(n-1)\Delta f$ is the radian frequency of the nth subcarrier, and the frequency spacing is $\Delta f = 1/T_s$.

For downlink transmission, all user signals are synchronously combined before transmission, hence they experience the same channel fading. Similar to an OFDM system, as long as the length of the cyclic prefix is equal to or larger than the maximum delay spread of the channel, ISI can be eliminated at the receiver by discarding the cyclic prefix. Therefore, the received signal $r(t)$ after cyclic prefix removal is

$$\begin{aligned} r(t) &= \eta(t) + \sum_{i=-\infty}^{\infty} \sqrt{\frac{2E_b}{N_c T_s}} \sum_{k=1}^{N_u} \sum_{n=1}^{N_c} h_n b_k(i) c_{k,n} \\ &\quad \cdot u_{T_s}(t - iT_s) \cos(w_n t + \varphi_n) \end{aligned} \tag{2}$$

where h_n is the subcarrier flat fading gain, φ_n is the subcarrier fading phase ($\{h_n, \varphi_n\}$ are common to all users), and $\eta(t)$ is zero-mean AWGN with single-sided power spectral density N_0. Assuming that coherent reception is employed, the channel amplitudes and phases $\{h_n, \varphi_n\}_{n=1}^{N_c}$ of all subcarriers are perfectly known to the receiver [2], [5], [8]. After phase compensation, the receiver performs amplitude correction described by $\alpha_n\,(n = 1, \cdots, N_c)$, which is also called equalizer coefficient. In the literature, different equalizer coefficient expressions for MMSEC have been proposed for MC-CDMA systems [13]–[15]. We will next derive the equalizer coefficient expressions for MMSEC and show how its computational complexity can be reduced.

After coherent demodulation (but before despreading and amplitude equalization), the received signal on the nth subcarrier is given by

$$y_n = \int_0^{T_s} r(t) \cos(w_n t + \varphi_n) dt = D_n + \mathcal{MUI}_n + \eta_n \tag{3}$$

where the desired signal and multi-user interference components are defined, respectively, as

$$D_n = \sqrt{\frac{E_bT_s}{2N_c}}\, b_1 c_{1,n} h_n \,, \tag{4}$$

$$\mathcal{MUI}_n = \sqrt{\frac{E_bT_s}{2N_c}}\, h_n \sum_{k=2}^{N_u} b_k c_{k,n} \,, \tag{5}$$

and the noise component η_n has zero mean and variance $N_0T_s/4$.

Denoting $\alpha_n \times c_{1,n}$, $\boldsymbol{\alpha'} = [\alpha'_1, \alpha'_2, \cdots, \alpha'_{N_c}]$, $\mathbf{y} = [y_1, y_2, \cdots, y_{N_c}]^T$. After equalization and despreading, the decision variable $U = \mathbf{y}^T\boldsymbol{\alpha'}$. The MMSEC scheme aims to find $\boldsymbol{\alpha'}$ that minimize the mean square error between U and b_1, i.e.,

$$\boldsymbol{\alpha'} = \arg\ \min_{\boldsymbol{\alpha'}} |b_1 - U|^2 \,. \tag{6}$$

The solution of the equation above is

$$\boldsymbol{\alpha'} = \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{R}_{b_1\mathbf{y}} \,. \tag{7}$$

Let $\mathbf{b} = [b_1, b_2, \cdots, b_{N_u}]^T$, $\boldsymbol{\eta} = [\eta_1, \eta_2, \cdots, \eta_{N_c}]^T$, $\boldsymbol{\mathcal{C}_d}$ is a $N_c \times N_u$ matrix with kth column being the spreading code for kth user, and $\boldsymbol{\mathcal{H}}$ is a diagonal matrix with nth diagonal element equals to h_n. We have

$$\mathbf{y} = \sqrt{\frac{E_bT_s}{2N_c}}\, \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}\mathbf{b} + \boldsymbol{\eta} \,. \tag{8}$$

Then the matrix $\mathbf{R}_{b_1\mathbf{y}}$ is

$$\begin{aligned}\mathbf{R}_{b_1\mathbf{y}} &= E\left[b_1 \times \left(\sqrt{\frac{E_bT_s}{2N_c}}\, \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}\mathbf{b} + \boldsymbol{\eta}\right)\right] \\ &= \sqrt{\frac{E_bT_s}{2N_c}} E[b_1\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}\mathbf{b}] \\ &= \sqrt{\frac{E_bT_s}{2N_c}}\, \boldsymbol{\mathcal{H}}\mathbf{c_1}\end{aligned} \tag{9}$$

where $\mathbf{c_1}$ is the first column of $\boldsymbol{\mathcal{C}_d}$.

The matrix $\mathbf{R}_{\mathbf{yy}}$ is

$$\begin{aligned}\mathbf{R}_{\mathbf{yy}} &= E\left[\left(\sqrt{\frac{E_bT_s}{2N_c}}\, \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}\mathbf{b} + \boldsymbol{\eta}\right)\left(\sqrt{\frac{E_bT_s}{2N_c}}\, \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}\mathbf{b} + \boldsymbol{\eta}\right)^T\right] \\ &= \frac{E_bT_s}{2N_c} E\left[\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}\mathbf{b}\mathbf{b}^T{\boldsymbol{\mathcal{C}_d}}^T\boldsymbol{\mathcal{H}}^T\right] + E\left[\boldsymbol{\eta} \times \boldsymbol{\eta}^T\right] \\ &= \frac{E_bT_s}{2N_c}\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}{\boldsymbol{\mathcal{C}_d}}^T\boldsymbol{\mathcal{H}} + \frac{N_0T_s}{4}\mathbf{I}_{N_c}\end{aligned} \tag{10}$$

where $\mathbf{I}_{N_c}$ is an identity matrix with size $N_c \times N_c$. Substituting (9) and (10) into (7), and taking note that $\mathbf{c_1}$ is the despreading code of the desired user 1, we obtain the set of equalizer coefficients for MMSEC scheme for downlink MC-CDMA as

$$\boldsymbol{\alpha} = \sqrt{\frac{2N_c}{E_bT_s}} \left(\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}{\boldsymbol{\mathcal{C}_d}}^T\boldsymbol{\mathcal{H}} + \frac{N_c}{2E_b/N_0}\mathbf{I}_{N_c}\right)^{-1} \cdot \mathbf{h} \tag{11}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_{N_c}]^T$ and $\mathbf{h} = [h_1, h_2, \cdots, h_{N_c}]^T$.

III. Complexity Reduction for MMSEC

The MMSEC equalizer coefficient vector $\boldsymbol{\alpha}$ was given in (11). The matrix to be inverted in (11) is

$$\begin{aligned}\mathbf{R} &= \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{C}_d}{\boldsymbol{\mathcal{C}_d}}^T\boldsymbol{\mathcal{H}} + \frac{N_c}{2E_b/N_0}\mathbf{I}_{N_c} \\ &= \begin{bmatrix} h_1^2x_{11} + \frac{N_c}{2E_b/N_0} & \cdots & h_1h_{N_c}x_{1N_c} \\ h_2h_1x_{21} & \cdots & h_{N_c}h_2x_{2N_c} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ h_{N_c}h_1x_{N_c1} & \cdots & h_{N_c}h_{N_c}x_{N_cN_c} + \frac{N_c}{2E_b/N_0} \end{bmatrix}\end{aligned} \tag{12}$$

where $x_{n,\,i}$ is the (n, i)th element of $\boldsymbol{\mathcal{C}_d}{\boldsymbol{\mathcal{C}_d}}^T$, i.e.,

$$x_{n,i} = \sum_{k=1}^{N_u} c_{kn}c_{ki} \,. \tag{13}$$

When $n = i$, we have $x_{n,\,n} = N_u$. This implies that the diagonal elements of $\boldsymbol{\mathcal{C}_d}{\boldsymbol{\mathcal{C}_d}}^T$ in $\mathbf{R}$ is always equals to N_u. If orthogonal spreading codes such as Walsh-Hadamard codes are used, the characteristics of $x_{n,\,i}$ will enable us to simplify the computation of $\mathbf{R}^{-1}$, and hence the corresponding MMSEC equalizer coefficient $\boldsymbol{\alpha}$.

We now consider the simplification of $\mathbf{R}^{-1}$ for different values of N_u.

Case 1: $N_u = N_c$ (full system loading condition)

In this case, due to the code orthogonality of Walsh-Hadamard codes, we have

$$x_{n,\,i} = 0\,, \quad \text{if} \quad n \neq i\,. \tag{14}$$

Hence $\mathbf{R}$ becomes a diagonal matrix, and the MMSEC equalizer coefficient in (11) can be reduced to

$$\boldsymbol{\alpha} = \left[\frac{h_1}{h_1^2 + \frac{1}{2}\frac{N_c}{N_u}\frac{1}{E_b/N_0}}, \quad \cdots \quad , \frac{h_{N_c}}{h_{N_c}^2 + \frac{1}{2}\frac{N_c}{N_u}\frac{1}{E_b/N_0}}\right]^T . \tag{15}$$

This implies that the nth equalizer coefficient only depends on the fading gain h_n of the nth subcarrier.

Case 2: $N_u = \frac{N_c}{2}$ (half system loading condition)

In this case, for Walsh-Hadamard codes, we have

$$x_{n,\,i} = \begin{cases} N_u\,, & \text{if} \quad n = i \quad \text{or} \quad |\, n - i \,| = N_c/2\,. \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{16}$$

Hence $\mathbf{R}$ has the form

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 \\ \mathbf{R}_3 & \mathbf{R}_4 \end{bmatrix} \tag{17}$$

where $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{R}_3$ and $\mathbf{R}_4$ are all diagonal matrices. For example,

$$\mathbf{R_1} = \begin{bmatrix} N_uh_1^2 + \frac{N_c}{2E_b/N_0} & \cdots & 0 \\ 0 & \cdots & 0 \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ 0 & \cdots & N_uh_{N_c/2}^2 + \frac{N_c}{2E_b/N_0} \end{bmatrix} . \tag{18}$$

$\mathbf{R}_2$, $\mathbf{R}_3$ and $\mathbf{R}_4$ have similar forms. In this case, it can be shown that the inverse of $\mathbf{R}$ is also composed of four diagonal matrices, i.e.,

$$\mathbf{R}^{-1} = \begin{bmatrix} \mathbf{R'}_1 & \mathbf{R'}_2 \\ \mathbf{R'}_3 & \mathbf{R'}_4 \end{bmatrix} \tag{19}$$

where $\mathbf{R'}_1$, $\mathbf{R'}_2$, $\mathbf{R'}_3$ and $\mathbf{R'}_4$ are all diagonal matrices. The detailed proof of (19) is given in Appendix I. Denoting the nth diagonal element of $\mathbf{R}_1$ as $r_{1,n}$, and the nth diagonal element of $\mathbf{R'}_1$ as $r'_{1,n}$, and so on, the elements of $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{R}_3$, $\mathbf{R}_4$ and the elements of $\mathbf{R'}_1$, $\mathbf{R'}_2$, $\mathbf{R'}_3$, $\mathbf{R'}_4$ are related by the following 2×2 matrix inversion

$$\begin{bmatrix} r'_{1,n} & r'_{2,n} \\ r'_{3,n} & r'_{4,n} \end{bmatrix} = \begin{bmatrix} r_{1,n} & r_{2,n} \\ r_{3,n} & r_{4,n} \end{bmatrix}^{-1}. \tag{20}$$

Therefore, we do not need to invert matrix $\mathbf{R}$ with size $N_c \times N_c$, which has complexity order $\mathcal{O}(N_c^3)$. With (20), we can obtain $\mathbf{R}^{-1}$ by simply inverting 2 by 2 matrices for $N_c/2$ times. Hence the complexity order is $\frac{N_c}{2}\mathcal{O}(2^3)$, which is much lower than $\mathcal{O}(N_c^3)$.

<u>Case 3</u>: $N_u = \frac{N_c}{4}$ or $N_u = \frac{3}{4}N_c$

In this case, it can be shown that the matrix $\mathbf{R}$ consists of 16 diagonal matrices, i.e.,

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 & \mathbf{R}_3 & \mathbf{R}_4 \\ \mathbf{R}_5 & \mathbf{R}_6 & \mathbf{R}_7 & \mathbf{R}_8 \\ \mathbf{R}_9 & \mathbf{R}_{10} & \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{13} & \mathbf{R}_{14} & \mathbf{R}_{15} & \mathbf{R}_{16} \end{bmatrix}. \tag{21}$$

$\mathbf{R}^{-1}$ is also composed of 16 diagonal matrices. Similar to the half system loading case, the elements of these diagonal matrices are related through

$$\begin{bmatrix} r'_{1,n} & r'_{2,n} & r'_{3,n} & r'_{4,n} \\ r'_{5,n} & r'_{6,n} & r'_{7,n} & r'_{8,n} \\ r'_{9,n} & r'_{10,n} & r'_{11,n} & r'_{12,n} \\ r'_{13,n} & r'_{14,n} & r'_{15,n} & r'_{16,n} \end{bmatrix} = \begin{bmatrix} r_{1,n} & r_{2,n} & r_{3,n} & r_{4,n} \\ r_{5,n} & r_{6,n} & r_{7,n} & r_{8,n} \\ r_{9,n} & r_{10,n} & r_{11,n} & r_{12,n} \\ r_{13,n} & r_{14,n} & r_{15,n} & r_{16,n} \end{bmatrix}^{-1}. \tag{22}$$

Therefore, for system loading at $N_u = \frac{N_c}{4}$ or $N_u = \frac{3}{4}N_c$, the elements of $\mathbf{R}^{-1}$ can be computed by the inversion of 4×4 matrices. As a result, the complexity of MMSEC is reduced to $\frac{N_c}{4}\mathcal{O}(4^3)$.

Similarly we can reduce the matrix for other system loading conditions. As a summary, Fig. 1 shows the relationship between the reduced complexity level and the system loading of MMSEC for an MC-CDMA system with 32 subcarriers. The horizontal axis represents the system loading ratio N_u/N_c while the vertical axis represents the complexity level in the form of $\mathcal{O}(y^3)$ (y is a dummy variable). Note that the complexity of the original MMSEC is $\mathcal{O}(N_c^3)$ or $\mathcal{O}(32^3)$ in this case. From Fig. 1, we can see that as long as the number of users is even, we can reduce the complexity to at least $\mathcal{O}((N_c/2)^3)$. If N_u/N_c is some special values, such as $1/2$ or $1/4$, we can reduce the complexity even more. Since the proposed technique of matrix inversion size reduction is derived mathematically without any approximation, the reduced-complexity MMSEC will have exactly the same output, hence exactly the same error rate performance, as the full-complexity MMSEC scheme. In addition, it is also worth taking note that the reduction of matrix size is independent of channel fading conditions.

<u>Case 4</u>: N_u is an odd number

If there are odd number of users in the system, however, we cannot reduce the matrix inversion size as before. But this problem can be circumvented easily. For example, by simply transmitting the signal of a dummy user, we can change N_u to an even number and make use of the method proposed earlier to reduce the matrix inversion size. However, the system performance is expected to be affected as one more interferer, the dummy user, is now present in the system. Fig. 2 shows the required E_b/N_0 for a 32-subcarrier MC-CDMA system with MMSEC to achieve BER=10^{-3} as N_u varies from 1 to 32 (full system loading condition). The channels considered are: a channel with 4 i.i.d. Rayleigh fading paths, and a channel with i.i.d. Rayleigh fading subcarriers. As seen from Fig. 2, around 0.2 dB more of E_b/N_0 is required to add one more user to the system while maintaining system performance at BER= 10^{-3}. This appears to be a reasonable trade-off to exchange for the complexity advantage of our proposed matrix inversion size reduction technique.

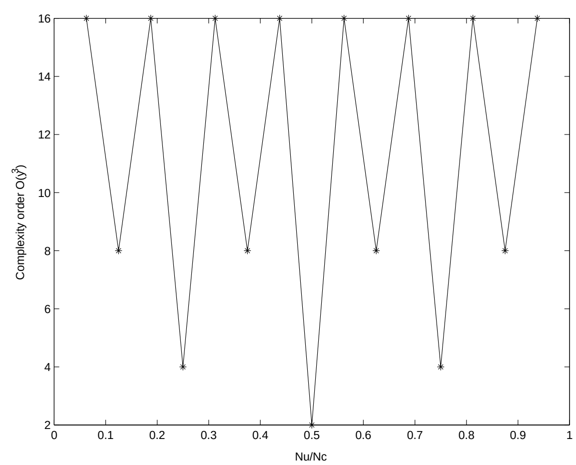

Fig. 1. Reduced complexity order vs. system loading for MC-CDMA system with 32 subcarriers.

IV. CONCLUSIONS

In this paper, we propose a method to reduce the computational complexity of the MMSEC equalization scheme for MC-CDMA downlink with orthogonal spreading. The complexity reduction is achieved by reducing the matrix inversion sizes. When the number of users N_u in the MC-CDMA system is even, we can reduce the matrix inversion size to at least $N_c/2$. When N_u/N_c has some special values, the complexity order can be reduced even more. For example, the complexity order is reduced to $\mathcal{O}(2^3)$ at $N_u/N_c = 1/2$ (half system loading condition). Complexity order is proportional to the number of multiplications and divisions required in a computation process, hence a lower complexity order directly implies lower computational complexity and implementation cost. This technique can be directly used for even number of

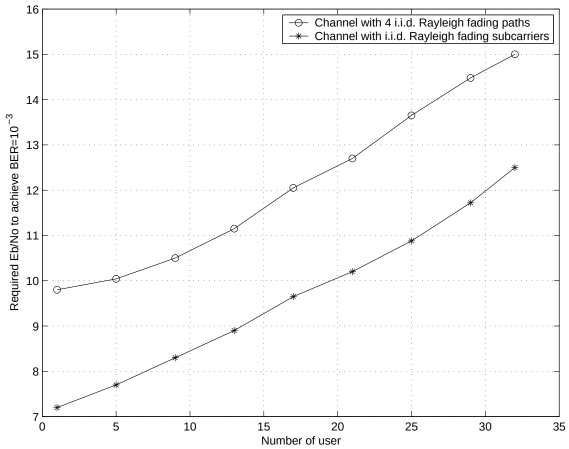

Fig. 2. Required E_b/N_0 to achieve BER=10^{-3} for MC-CDMA system with MMSEC in channels of 4 i.i.d. Rayleigh fading paths or of i.i.d. Rayleigh fading subcarriers.

N_u. If N_u is an odd number, by simply transmitting a dummy user, the system performance does not degrade much, while we can apply the proposed technique to reduce the matrix inversion size.

References

[1] A. C. McCormick and E. A. Al-Susa, "Multicarrier CDMA for future generation mobile communication," *Electronics & Communication Engineering Journal*, vol. 14, no. 2, pp. 52–60, Apr. 2002.

[2] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Commun. Mag.*, pp. 126–133, Dec. 1997.

[3] X. Gui and T. S. Ng, "Performance of asynchronous orthogonal multicarrier CDMA system in frequency selective fading channel," *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1084–1091, Jul. 1999.

[4] S. Hara and R. Prasad, "Design and performance of multi-carrier DS-CDMA system in multi-path fading channels," *IEEE Trans. Veh. Technol.*, vol. 48, no. 5, pp. 1584–1595, Sep. 1999.

[5] K. Fazel and S. Kaiser, *Multi-Carrier and Spread Spectrum Systems*. John Wiley and Sons: Chichester, 2003.

[6] S. L. Miller and B. J. Rainbolt, "MMSE detection of Multicarrier CDMA," *IEEE J. Select. Areas Commun.*, vol. 18, no. 11, pp. 2356–2362, Nov. 2000.

[7] W. T. Tung and J. Wang, "MMSE receiver for multicarrier CDMA overlay in ultra-wide-band communications," *IEEE Trans. Veh. Technol.*, vol. 54, no. 2, pp. 603–614, 2005.

[8] J. F. Helard, J. Y. Baudais, and J. Giterne, "Linear MMSE detection techniques for MC-CDMA," *IEE Electronic Lett.*, vol. 36, no. 7, pp. 665–666, Mar. 2000.

[9] O. Edfors, M. Sandel, J. J. van de Beek, S. K. Wilson, and P. O. Brjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931–939, Jul. 1998.

[10] M. H. Hsieh and C. H. Wei, "Channel Estimation for OFDM Systems based on Comb-Type Pilot Arrangement in Frequency Selective Fading Channels," *IEEE Trans. Consumer Electronics*, vol. 44, no. 1, pp. 217–225, Feb. 1998.

[11] L. Li, M. Tulino, and S. Verdu, "Design of reduced-rank MMSE multiuser detectors using random matrix methods," *IEEE Trans. Info. Theo.*, vol. 50, no. 6, pp. 986–1008, Jun. 2004.

[12] E. Strom and S. L. Miller, "A reduced complexity adaptive near-far resistant receiver for DS-CDMA," in *Proc. IEEE Global Commun. Conf. (GLOBECOM '93)*, Nov. 1993, pp. 1734–1738.

[13] S. Kaiser, "On the performance of different detection techniques for OFDM-CDMA in fading channels," in *Proc. IEEE Global Telecommunications Conf. (GLOBECOM'95)*, 1995, pp. 2059–2063.

[14] D. N. Kalofonos, M. Stojanovic, and J. G. Proakis, "On the performance of adaptive MMSE detectors for a MC-CDMA system in fast fading Rayleigh channels," in *IEEE Ninth Int. Symp. Personal, Indoor and Mobile Radio Communication (PIMRC'98)*, Sep. 1998, pp. 1309–1313.

[15] D. Mottier, D. Castelain, J. F. Herlard, and J. Y. Baudais, "Optimum and sub-optimum linear MMSE multi-user detection for multi-carrier CDMA transmission systems," in *IEEE Vehicular Technology Conference (VTC'01 FALL)*, 2001.

Appendix A
Calculation of $\mathbf{R}^{-1}$

The MMSE receiver used in this paper requires to compute $\mathbf{R}^{-1}$ (see (11) - (22)). In general, $\mathbf{R}$ can be expressed in a partitioned matrix form

$$\mathbf{R} = \begin{bmatrix} \mathbf{A}_{1,1} & \mathbf{A}_{1,2} & \dots & \mathbf{A}_{1,M} \\ \mathbf{A}_{2,1} & \mathbf{A}_{2,2} & \dots & \mathbf{A}_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{M,1} & \mathbf{A}_{M,2} & \dots & \mathbf{A}_{M,M} \end{bmatrix} \quad (23)$$

with

$$\mathbf{A}_{m,n} = \text{diag}\{A^{(1)}_{m,n}, A^{(2)}_{m,n}, \cdots, A^{(L)}_{m,n}, \}, m, n = 1, 2, \cdots, M. \quad (24)$$

We can see from (23) and (24) that $\mathbf{R}$ is a sparse matrix with a very special structure. Specifically, $\mathbf{R}$ is partitioned into M^2 diagonal matrices $\{\mathbf{A}_{m,n}\}$. Exploiting this structure, we can obtain another partitioned matrix given by

$$\widetilde{\mathbf{R}} = \text{diag}\{\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_L\} \quad (25)$$

with

$$\mathbf{B}_l = \begin{bmatrix} A^{(l)}_{1,1} & A^{(l)}_{1,2} & \dots & A^{(l)}_{1,M} \\ A^{(l)}_{2,1} & A^{(l)}_{2,2} & \dots & A^{(l)}_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ A^{(l)}_{M,1} & A^{(l)}_{M,2} & \dots & A^{(l)}_{M,M} \end{bmatrix}, l = 1, 2, \cdots, L. \quad (26)$$

For convenience, we define transforms between $\mathbf{R}$ and $\widetilde{\mathbf{R}}$ by the following canonical operators

Definition 1 ($\mathbf{R} \rightarrow \widetilde{\mathbf{R}}$):

$$\widetilde{\mathbf{R}} \triangleq \mathcal{L}(\mathbf{R}), \quad (27)$$

Definition 2 ($\widetilde{\mathbf{R}} \rightarrow \mathbf{R}$):

$$\mathbf{R} \triangleq \mathcal{L}^{-1}(\widetilde{\mathbf{R}}). \quad (28)$$

The operator $\mathcal{L}(\cdot)$ can be implemented by permutating columns and rows of $\mathbf{R}$ or, equivalently, by picking up the lth diagonal element in $\mathbf{A}_{m,n}$ to construct $\mathbf{B}_l$ ($l = 1, 2, \cdots, L$). Before proceeding further, we need the following *lemma*.

Lemma 1: The inverse of $\mathbf{R}$, $\mathbf{R}^{-1}$, has the same structure as described by (23) and (24). In other words, $\mathbf{R}^{-1}$ is a partitioned sparse matrix composed of M^2 diagonal matrices of size $L \times L$.

Proof: We prove this *lemma* via the induction principle.

(1) $M = 1$

When $M = 1$, this is a trivial case, since $\mathbf{R}$ reduces to a diagonal matrix.

(2) $M = 2$

When $M = 2$, $\mathbf{R}$ can be written as

$$\mathbf{R}(2) = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \quad (29)$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are diagonal matrices. According to the well-known equation

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{B}\mathbf{E}\mathbf{C}\mathbf{A}^{-1} & -\mathbf{A}^{-1}\mathbf{B}\mathbf{E} \\ -\mathbf{E}\mathbf{C}\mathbf{A}^{-1} & \mathbf{E} \end{bmatrix} \tag{30}$$

where we assume $\mathbf{A}^{-1}$ and $\mathbf{E} \triangleq (\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1}$ exist, it is relatively straightforward to verify that $\mathbf{R}^{-1}(2)$ comprises 4 diagonal sub-matrices.

(3) $M = m \Longrightarrow M = m + 1$

Let

$$\mathbf{R}(m) \triangleq \begin{bmatrix} \mathbf{A}_{1,1} & \mathbf{A}_{1,2} & \dots & \mathbf{A}_{1,m} \\ \mathbf{A}_{2,1} & \mathbf{A}_{2,2} & \dots & \mathbf{A}_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{m,1} & \mathbf{A}_{m,2} & \dots & \mathbf{A}_{m,m} \end{bmatrix}. \tag{31}$$

Assume that *lemma* 1 holds for $M = m$, i.e.,

$$\mathbf{R}^{-1}(m) = \begin{bmatrix} \mathbf{A}'_{1,1} & \mathbf{A}'_{1,2} & \dots & \mathbf{A}'_{1,m} \\ \mathbf{A}'_{2,1} & \mathbf{A}'_{2,2} & \dots & \mathbf{A}'_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}'_{m,1} & \mathbf{A}'_{m,2} & \dots & \mathbf{A}'_{m,m} \end{bmatrix} \tag{32}$$

where all sub-matrices $\mathbf{A}_{m,n}$ and $\mathbf{A}'_{m,n}$ are diagonal. We consider the case of $M = m + 1$. Similar to (29), $\mathbf{R}$ can now be written in a partitioned matrix form

$$\begin{aligned} \mathbf{R}(m+1) &\triangleq \begin{bmatrix} \mathbf{A}_{1,1} & \mathbf{A}_{1,2} & \dots & \mathbf{A}_{1,m+1} \\ \mathbf{A}_{2,1} & \mathbf{A}_{2,2} & \dots & \mathbf{A}_{2,m+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{m+1,1} & \mathbf{A}_{m+1,2} & \dots & \mathbf{A}_{m+1,m+1} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \end{aligned} \tag{33}$$

where

$$\begin{aligned} \mathbf{A} &= \mathbf{R}(m) \\ \mathbf{B} &= \begin{bmatrix} \mathbf{A}_{1,m+1} \\ \mathbf{A}_{2,m+1} \\ \vdots \\ \mathbf{A}_{m,m+1} \end{bmatrix} \\ \mathbf{C} &= [\mathbf{A}_{m+1,1}, \mathbf{A}_{m+1,2}, \cdots, \mathbf{A}_{m+1,m}] \\ \mathbf{D} &= \mathbf{A}_{m+1,m+1} \end{aligned} \tag{34}$$

Invoking (30), we can see that $\mathbf{R}^{-1}(m+1)$ is composed of $(m+1)^2$ diagonal matrices. ■

With *lemma* 1 at hand, we have the main result of this appendix.

Theorem 1: The inverse of $\mathbf{R}$, which is described by (23) and (24), can be calculated as

$$\mathbf{R}^{-1} = \mathcal{L}^{-1}\left(\widetilde{\mathbf{R}}^{-1}\right) = \mathcal{L}^{-1}\left(\left[\mathcal{L}\left(\mathbf{R}\right)\right]^{-1}\right) \tag{35}$$

where the inverse of $\widetilde{\mathbf{R}}$, which is described by (25), can be readily given by

$$\widetilde{\mathbf{R}}^{-1} = \mathrm{diag}\{\mathbf{B}_1^{-1}, \mathbf{B}_2^{-1}, \cdots, \mathbf{B}_L^{-1}\}. \tag{36}$$

Proof: Suppose there are two length-ML columns vectors $\mathbf{x}$ and $\mathbf{y}$, which satisfy

$$\mathbf{R}\mathbf{x} = \mathbf{y}. \tag{37}$$

Accordingly, we have

$$\mathbf{x} = \mathbf{R}^{-1}\mathbf{y}. \tag{38}$$

We define $\widetilde{\mathbf{x}} \triangleq \mathcal{L}(\mathbf{x})$ and $\widetilde{\mathbf{y}} \triangleq \mathcal{L}(\mathbf{y})$. Note that only row permutations are needed if the operator $\mathcal{L}(\cdot)$ acts on column vectors. Applying the operator $\mathcal{L}(\cdot)$ to (37), we obtain

$$\widetilde{\mathbf{R}}\widetilde{\mathbf{x}} = \widetilde{\mathbf{y}} \tag{39}$$

which yields

$$\widetilde{\mathbf{x}} = \widetilde{\mathbf{R}}^{-1}\widetilde{\mathbf{y}}. \tag{40}$$

Similarly, because the operator $\mathcal{L}(\cdot)$ exerts exactly the same effect on $\mathbf{R}$ and $\mathbf{R}^{-1}$ (due to *lemma* 1), we obtain from (38)

$$\widetilde{\mathbf{x}} = \mathcal{L}\left(\mathbf{R}^{-1}\right)\widetilde{\mathbf{y}}. \tag{41}$$

By comparing (40) and (41), it follows directly that

$$\mathcal{L}\left(\mathbf{R}^{-1}\right) = \widetilde{\mathbf{R}}^{-1} \Longrightarrow \mathbf{R}^{-1} = \mathcal{L}^{-1}\left(\widetilde{\mathbf{R}}^{-1}\right). \tag{42}$$

■

Returning to the MC-CDMA system discussed in this paper, we have $ML = N_c$ and the value of M or L depends on the number of users N_u.